HEAVY TETRAQUARKS

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Ebert, Faustov, Galkin (EFG) — Phys. Lett. B (2006) 214-219 [hep-ph/0512230]
Ebert, Faustov, Galkin (EFG) — arXiv:0808.3912 [hep-ph]

New Charmonium-like Mesons

Belle, BaBar, CDF, D0, CLEO

X(3872)

X(3876)

Y(3940)

Z(4051)

X(4160)

Z(4248)

Y(4260)

Y(4360)

Z(4430)

Y(4660)

TETRAQUARKS in diquark-antidiquark picture

Tetraquarks — diquark and antidiquark in colour $\bar{3}$ and 3 configurations bound by colour forces $\Longrightarrow$

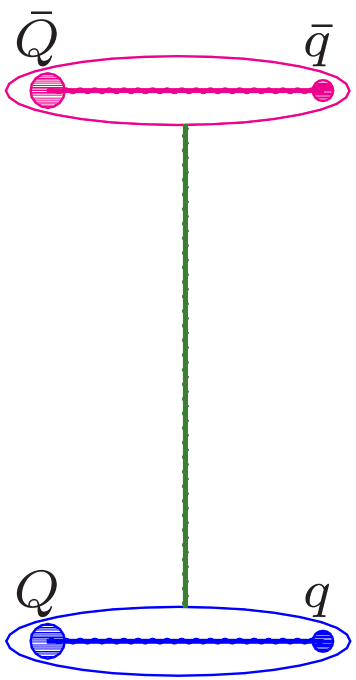
⋆ typical hadronic size

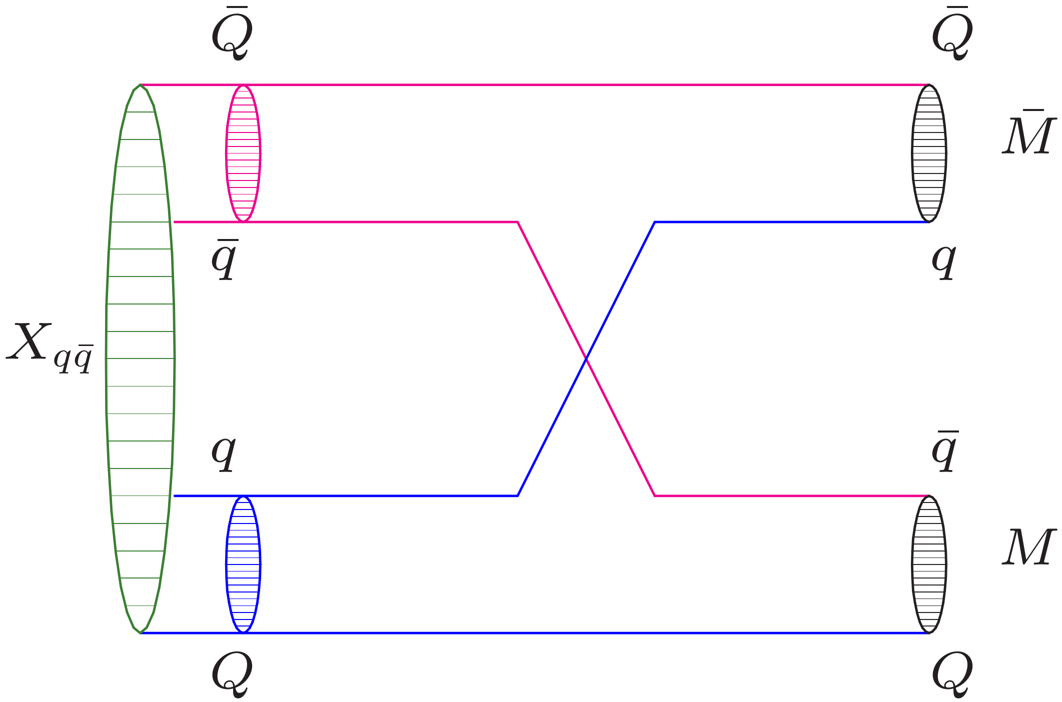
⋆ X should be split into two states $[cu][\bar{c}\bar{u}]$ and $[cd][\bar{c}\bar{d}]$ with $\Delta M \sim 7$ MeV

⋆ existence of charged partners $X^+ = [cu][\bar{c}\bar{d}]$, $X^- = [cd][\bar{c}\bar{u}]$

⋆ existence of tetraquarks with open $X_{s\bar{q}} = [cs][\bar{c}\bar{q}]$ and hidden $X_{s\bar{s}} = [cs][\bar{c}\bar{s}]$ strangeness

⋆ rich spectroscopy — radial and orbital excitations between diquarks





$(Q = c, b \qquad q = u, d$ or $s)$

Both scalar (antisymmetric in flavour $[cq]_{S=0} = [cq]$) and axial vector (symmetric in flavour $[cq]_{S=1} = \{cq\}$) diquarks are considered

The $[cq][\bar{c}\bar{q}']$ ground states:

- ⋆ Two states with $J^{PC} = 0^{++}$:
  (C is defined for $q = q'$)

$$X(0^{++}) = [cq]_{S=0}[\bar{c}\bar{q}']_{S=0}$$

$$X(0^{++\prime}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ⋆ Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}([cq]_{S=1}[\bar{c}\bar{q}']_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}']_{S=1})$$

$$X(1^{+-}) = \frac{1}{\sqrt{2}}([cq]_{S=0}[\bar{c}\bar{q}']_{S=1} - [cq]_{S=1}[\bar{c}\bar{q}']_{S=0})$$

$$X(1^{+-\prime}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ⋆ One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

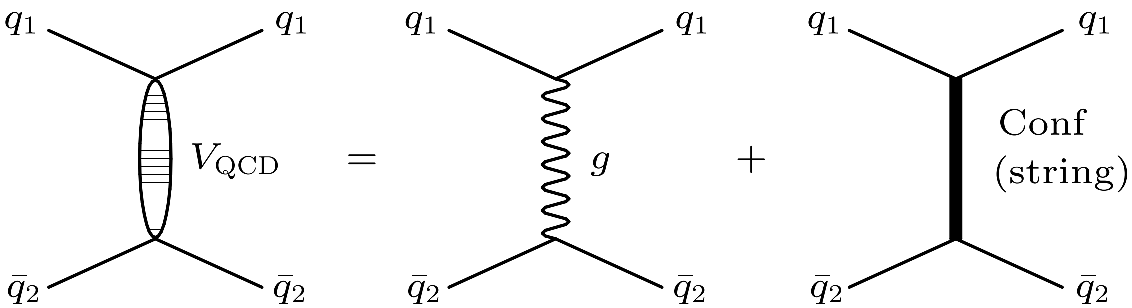
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector

- $q\bar{q}$ **quasipotential**



$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu+V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu}+V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k}=\mathbf{p}-\mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^{\lambda}(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \left(\begin{array}{c} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{array} \right) \chi^{\lambda},$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V_{\rm conf}^{V} + V_{\rm conf}^{S}$

In nonrelativistic limit

$$\left.\begin{array}{rcl} V_{\rm conf}^{V} & = & (1-\varepsilon)(Ar+B) \\ V_{\rm conf}^{S} & = & \varepsilon(Ar+B) \end{array}\right\} \quad \text{Sum}: \quad (Ar+B)$$

ε - mixing parameter

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- **Heavy tetraquarks in diquark-antidiquark picture**

(Qq)-interaction: $V_{Qq} = \frac{1}{2}V_{Q\bar{q}}$

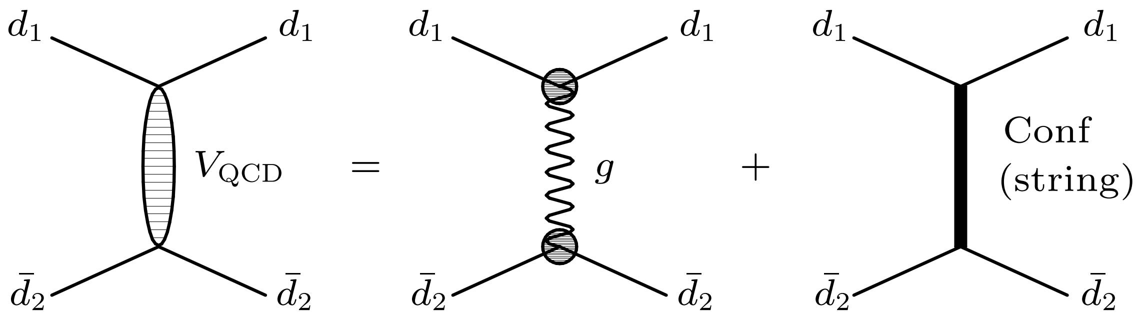
$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + \frac{1}{2}V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + \frac{1}{2}V^S_{\rm conf}(\mathbf{k})$$

$(d_1\bar{d}_2)$-interaction: $\quad d = (Qq)$

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d_1(P)|J_\mu|d_1(Q)\rangle}{2\sqrt{E_{d_1}E_{d_1}}}\frac{4}{3}\alpha_S D^{\mu\nu}(\mathbf{k})\frac{\langle d_2(P')|J_\nu|d_2(Q')\rangle}{2\sqrt{E_{d_2}E_{d_2}}}$$
$$+\psi^*_{d_1}(P)\psi^*_{d_2}(P')\left[J_{d_1;\mu}J^{\mu}_{d_2}V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right]\psi_{d_1}(Q)\psi_{d_2}(Q'),$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for scalar diquark} \\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark} \\ & (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

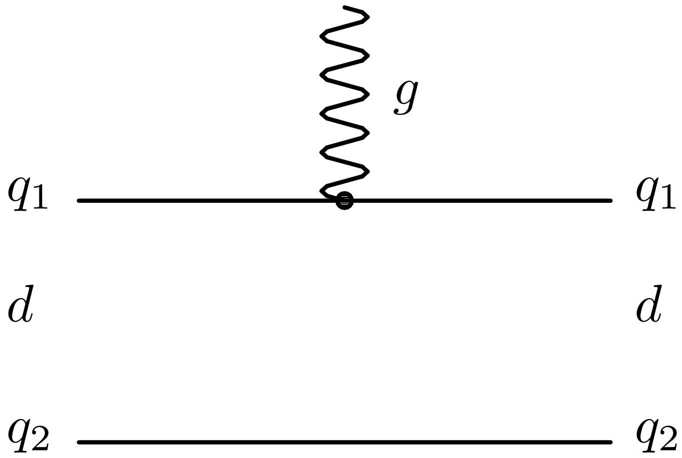


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

DIQUARKS

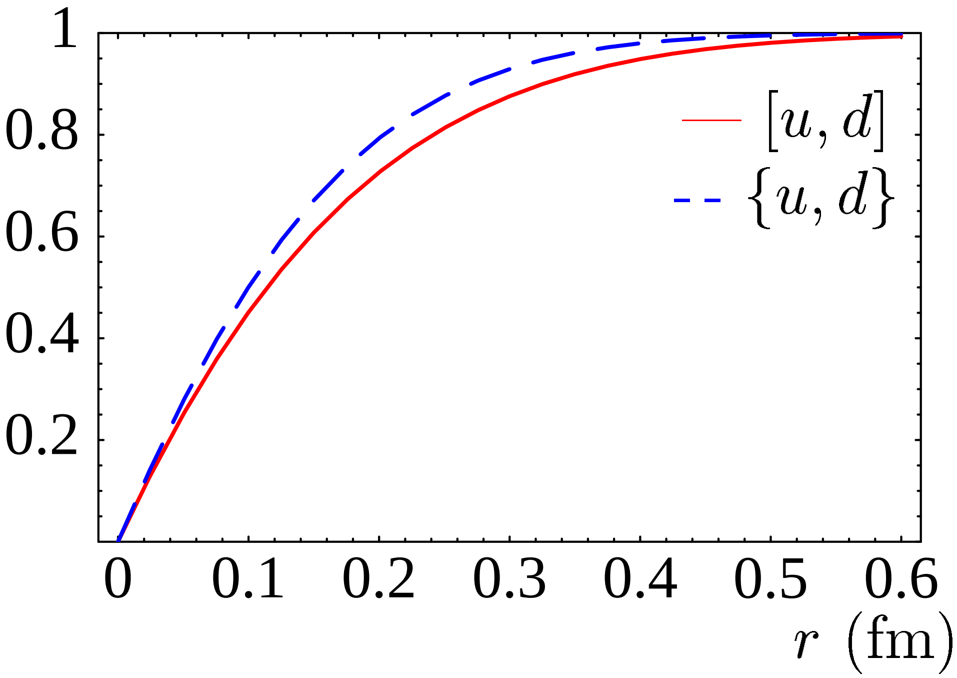
Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE	Maris BSE	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

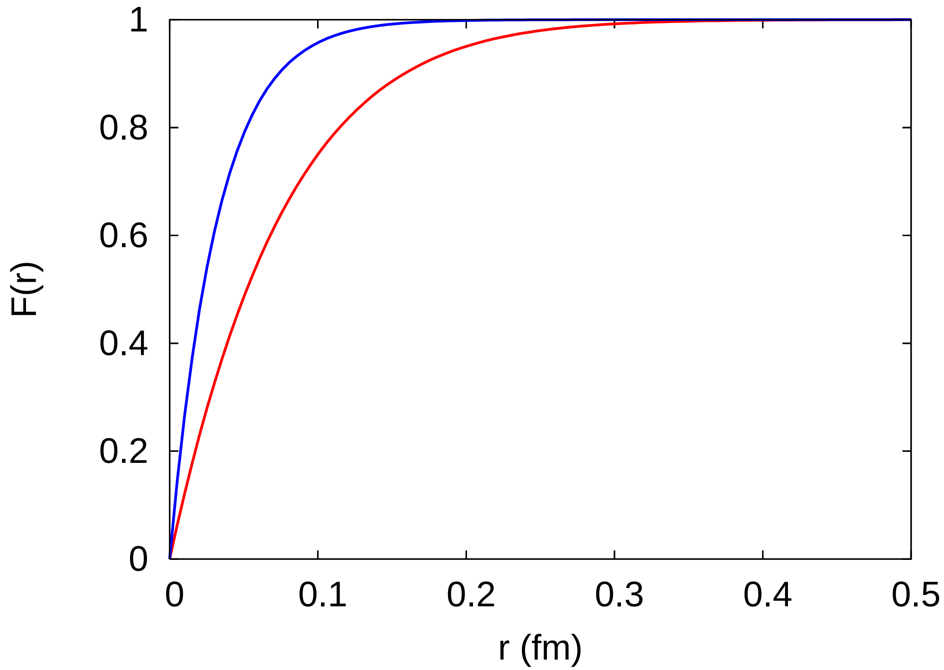
Table 2: Masses of heavy-light and doubly heavy diquarks (MeV).

Quark content	Diquark type	Mass	
		$Q = c$	$Q = b$
$[Q, q]$	S	1973	5359
$\{Q, q\}$	A	2036	5381
$[Q, s]$	S	2091	5462
$\{Q, s\}$	A	2158	5482
$[Q, c]$	S		6519
$\{Q, c\}$	A	3226	6526
$\{Q, b\}$	A	6526	9778

The form factors $F(r)$ for the scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks:



The form factors $F(r)$ for $\{c, q\}$ (red line) and $\{b, q\}$ (blue line) axial vector diquarks.



The potential of the heavy diquark-antidiquark interaction

$$V(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{2}\Bigg\{\left[\frac{1}{E_1(E_1+M_1)} + \frac{1}{E_2(E_2+M_2)}\right]\frac{\hat{V}'_{\rm Coul}(r)}{r} - \left[\frac{1}{M_1(E_1+M_1)}\right.$$
$$\left. + \frac{1}{M_2(E_2+M_2)}\right]\frac{V'_{\rm conf}(r)}{r} + \frac{\mu_d}{2}\left(\frac{1}{M_1^2}+\frac{1}{M_2^2}\right)\frac{V'^V_{\rm conf}(r)}{r}\Bigg\}\mathbf{L}\cdot(\mathbf{S}_1+\mathbf{S}_2)$$
$$+\frac{1}{2}\Bigg\{\left[\frac{1}{E_1(E_1+M_1)} - \frac{1}{E_2(E_2+M_2)}\right]\frac{\hat{V}'_{\rm Coul}(r)}{r} - \left[\frac{1}{M_1(E_1+M_1)} - \frac{1}{M_2(E_2+M_2)}\right]\frac{V'_{\rm conf}(r)}{r}$$
$$+\frac{\mu_d}{2}\left(\frac{1}{M_1^2}-\frac{1}{M_2^2}\right)\frac{V'^V_{\rm conf}(r)}{r}\Bigg\}\mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2) + \frac{1}{E_1E_2}\Bigg\{\mathbf{p}\left[V_{\rm Coul}(r)+V^V_{\rm conf}(r)\right]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r)$$
$$+V'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r} + \frac{1}{r}\left[V'_{\rm Coul}(r) + \frac{\mu_d}{4}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)V'^V_{\rm conf}(r)\right]\mathbf{L}(\mathbf{S}_1+\mathbf{S}_2)$$
$$+\frac{1}{3}\left[\frac{1}{r}V'_{\rm Coul}(r) - V''_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^V_{\rm conf}(r) - V''^V_{\rm conf}(r)\right)\right]\left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right]$$
$$+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}-\frac{E_2}{M_2}\right)\frac{V'^V_{\rm conf}(r)}{r}\mathbf{L}(\mathbf{S}_1-\mathbf{S}_2) + \frac{2}{3}\left[\Delta V_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_1\mathbf{S}_2\Bigg\},$$

where

$$V_{\rm Coul}(r) = -\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

The diquark-antidiquark model of heavy tetraquarks predicts existence of the $SU(3)$ nonet of states with hidden charm or beauty ($Q = c, b$):

• four tetraquarks ($[Qq][\bar{Q}\bar{q}]$, $q = u, d$) with neither open nor hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1

• four tetraquarks ($[Qs][\bar{Q}\bar{q}]$ and $[Qq][\bar{Q}\bar{s}]$, $q = u, d$) with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$

• one tetraquark ($[Qs][\bar{Q}\bar{s}]$) with hidden strangeness and zero electric charge.

In our model we neglect the mass difference of u and d quarks and electromagnetic interactions – thus corresponding tetraquarks will be degenerate in mass. More detailed analysis predicts that such mass differences can be of few MeV.

The (non)observation of such states will be a crucial test of the tetraquark model.

Table 3: Masses of hidden charm tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cs\bar{c}\bar{q}/cq\bar{c}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080
$1P$				
1^{--}	$S\bar{S}$	4244	4466	4350

Table 4: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta' J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

Table 5: Masses of hidden bottom tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass		
		$bq\bar{b}\bar{q}$	$bs\bar{b}\bar{s}$	$bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	10471	10662	10572
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	10492	10682	10593
0^{++}	$A\bar{A}$	10473	10671	10584
1^{+-}	$A\bar{A}$	10494	10686	10599
2^{++}	$A\bar{A}$	10534	10716	10628
$1P$				
1^{--}	$S\bar{S}$	10807	11002	10907

Table 6: Thresholds for open bottom decays.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$B\bar{B}$	10558	$B_s^+ B_s^-$	10739	BB_s	10649
$B\bar{B}^*$	10604	$B_s^\pm B_s^{*\mp}$	10786	$B^* B_s$	10695
$B^*\bar{B}^*$	10650	$B_s^{*+} B_s^{*-}$	10833	$B^* B_s^*$	10742

Table 7: Masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV).

State	Diquark	Theory			Experiment	
J^{PC}	content	EFG	Maiani	Maiani ($cs\bar{c}\bar{s}$)	state	mass
$1S$						
0^{++}	$S\bar{S}$	3812	3723			
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871	3872†		$X(3872)$ $X(3876)$	3871.4 ± 0.6 $3875.2 \pm 0.7^{+0.9}_{-1.8}$
1^{+-}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	3871	3754			
0^{++}	$A\bar{A}$	3852	3832			
1^{+-}	$A\bar{A}$	3890	3882			
2^{++}	$A\bar{A}$	3968	3952		$Y(3943)$	$3943 \pm 11 \pm 13$ $3914.3^{+4.1}_{-3.8}$
$1P$						
1^{--}	$S\bar{S}$	4244		4330±70	$Y(4260)$	$4259 \pm 8^{+2}_{-6}$ $4247 \pm 12^{+17}_{-32}$
1^{-}	$S\bar{S}$	4244			$Z(4248)$	$4248^{+44+180}_{-29-35}$
0^{-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4267				
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284			$Y(4260)$	$4284^{+17}_{-16}\pm4$
1^{--}	$A\bar{A}$	4277				
1^{--}	$A\bar{A}$	4350			$Y(4360)$	$4361 \pm 9 \pm 9$ 4324 ± 24
$2S$						
1^{+}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431			$Z(4430)$	$4433\pm4\pm2$
0^{+}	$A\bar{A}$	4434				
1^{+}	$A\bar{A}$	4461	$\sim$ 4470			
$2P$						
1^{--}	$S\bar{S}$	4666			$Y(4660)$ $X(4630)$	$4664 \pm 11 \pm 5$ 4634^{+8+5}_{-7-8}

† input

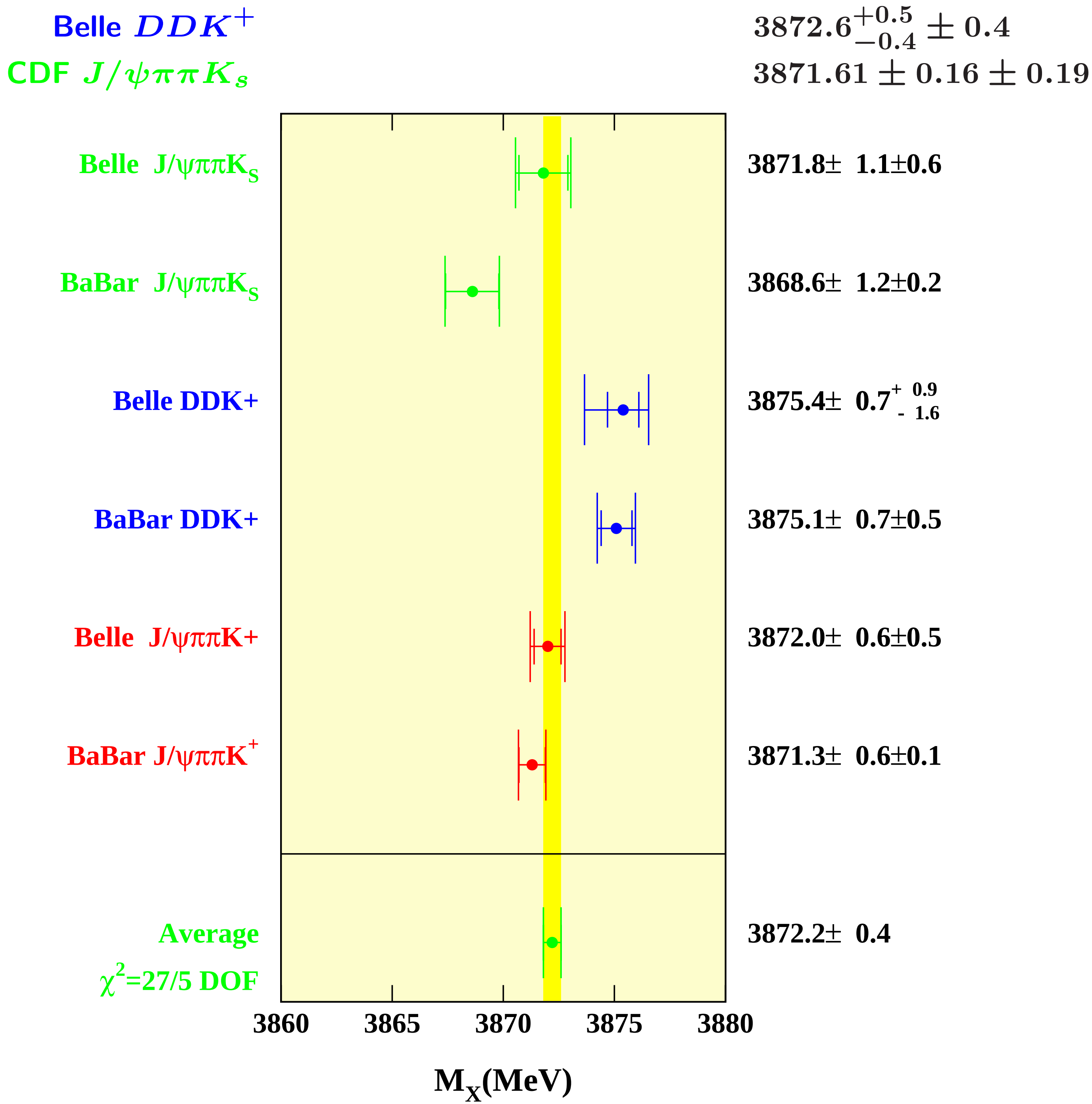


Figure 2: Measured masses of the $X(3872)$

P.D. Jackson, arXiv:0806.3654 [hep-ex]

SUMMARY

- Masses of heavy tetraquarks with hidden and open charm and bottom are calculated in the diquark-antidiquark picture.
- Dynamical approach based on the relativistic quark model is used, where both diquark and tetraquark masses are obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials.
- The diquark size is taken into account with the help of the diquark-gluon form factor in terms of diquark wave functions.
- No free adjustable parameters are introduced.
- $X(3872)$ can be the 1^{++} neutral charm tetraquark state. If it is really a tetraquark, one more neutral and two charged tetraquark states should exist with close masses.
- $Y(4260)$, $Y(4360)$ and $Y(4660)$ can be the 1^{--} P-wave tetraquark states.
- Charged $Z(4433)$ can be the 1^{+} or 0^{+} $2S$-wave tetraquark state.
- Charged $Z(4248)$ can be the 1^{-} or 0^{-} $1P$-wave tetraquark state.
- The ground states of tetraquarks with hidden bottom are predicted to have masses below the open bottom threshold and thus should be narrow.