Chiral transitions in heavy–light mesons

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published in PRD **77**, 074017 (2008)

Introduction

In most cases the quark model predicts meson states in good agreement with experiment. And we believe that we have a theory of bound states in QCD, working well in general. However, there are exclusions. Some states are shifted from the places prescribed by this theory. Some examples for mesons:

a) pions and kaons

b) some heavy-light mesons, like D_s

c) some light scalar mesons, like a_0, f_0.

In baryons, examples include $N(1440)(Roper), \Lambda(1405)$, etc. We expect that in some of these cases the Channel Coupling interaction is operating and explains the strong shift of masses. Moreover, we suggest a new mechanism, called the chiral coupling of channels, which works for a bound light quark and can explain the shift of $\gtrsim$ 100 MeV in a mass. It is important, that this new mechanism does not contain any fitting parameters and its predictions are fixed in some sense.

We start from the light quark + meson effective Lagrangian

$$\boxed{L = i\int d^4x d^4y \psi^+(x)\widehat{M}(x,y)\psi(y)}$$

which includes both effect of confinement and chiral symmetry breaking.
Here

$$\boxed{\widehat{M}(x,y) = M_S(x,y)e^{i\gamma_5\widehat{\phi}(x,y)}}$$

and in the limit of small $T_g \to 0$

$$\boxed{M_S(x,y) \approx \sigma|\mathbf{x}|\delta^{(4)}(x-y), |\mathbf{x}| \gg T_g}$$

To lowest order in $\widehat{\phi}$

$$\boxed{\Delta L^{(1)} = \int \overline{\psi}(x)\sigma|\mathbf{x}|\gamma_5 \frac{\pi^A\lambda^A}{F_\pi}\psi(x)dtd^3\mathbf{x}}$$

$$\pi^A\lambda^A = \sqrt{2}\begin{pmatrix} \frac{\eta^0}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & K^+ \\ \pi^-, & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}_0, & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix}$$

For chiral transitions between heavy-light mesons we obtain the matrix element

$$U_{21} = \int \overline{\psi}_2(\mathbf{x})\frac{\sigma|\mathbf{x}|}{F_\pi}\gamma_5\frac{\lambda^A e^{i\mathbf{kx}}}{\sqrt{2\omega_\pi(\mathbf{k})V}}\psi_1(\mathbf{x})d^3\mathbf{x}$$

and the corresponding perturbation width

$$w = 2\pi|U_{21}|^2\delta(E_1 - E_2 - \omega)\frac{Vd^3\mathbf{k}}{(2\pi)^3}$$

Let $\widehat{U}$ be any (local or nonlocal) operator, which causes a transition from channel i to channel j by the matrix element U_{ij}. Then the multichannel relativistic Green's function G_{ik} satisfies Hamiltonian-like equations (N – number of channels):

$$[(H_i - M)\delta_{ik} + U_{il}]G_{lk} = 1, \quad i,k,l = 1,...,N$$

For two channels one can reduce the problem to an effective one-channel equation

$$(H_1 - E)G_{11} - U_{12}\frac{1}{H_2 - E}U_{21}G_{11} = 1$$

Consider an unperturbed set of states $|n\rangle$ in channel 1 and $|m\rangle$ in channel 2. Then a nonlinear equation for the energy eigenvalue M is

$$M = M_1^{(n)} - \sum_m \langle n|U_{12}|m\rangle \frac{1}{M_2^{(m)} - M}\langle m|U_{21}|n\rangle$$

Calculation details

Equation for the mass shift of the i^{th} state:

$$m[i] = m^{(0)}[i] - \sum_f \frac{|<i|\hat{V}|f>|^2}{E_f - m[i]}$$

We consider the following thresholds:

i	$D_s(0^+)$	$D_s(1^+)$	$D_s(2^+)$
f	$D(0^-) + K(0^-)$	$D^*(1^-) + K(0^-)$	$D^*(1^-) + K(0^-)$

i	$B_s(0^+)$	$B_s(1^+)$	$B_s(2^+)$
f	$B(0^-) + K(0^-)$	$B^*(1^-) + K(0^-)$	$B^*(1^-) + K(0^-)$

Masses of meson states used for calculation (MeV):

$$m_D = 1869, \quad m_D + m_K = 2363$$

$$m_{D^*} = 2010, \quad m_{D^*} + m_K = 2504$$

$$m_B = 5279, \quad m_B + m_K = 5773$$

$$m_{B^*} = 5325, \quad m_{B^*} + m_K = 5819$$

In what follows all equations are given for D, D_s mesons; changes for B, B_s mesons are obvious.

Notations:

$$E_f = \omega_D + \omega_K, \quad T_f = E_f - m_D - m_K,$$

$$\omega_K(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_K^2}, \quad \omega_D(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_D^2},$$

$$E_0 = m^{(0)}[D_s] - m_D - m_K, \quad \delta m = m[D_s] - m^{(0)}[D_s],$$

$$\Delta = E_0 + \delta m = m[D_s] - m_D - m_K$$

We neglect the DK interacting and the D-meson moving in the f-state, so

$$|f> = \Psi_K(\mathbf{p}) \otimes \Psi_D(M_f), \quad |i> = \Psi_{D_s}(M_i),$$

$$\Psi_K(\mathbf{p}) = \frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}, \quad \sum_f \to \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{M_f}$$

One may write the equation for the mass shift as

$$\boxed{E_0 - \Delta = \mathcal{F}(\Delta)}$$

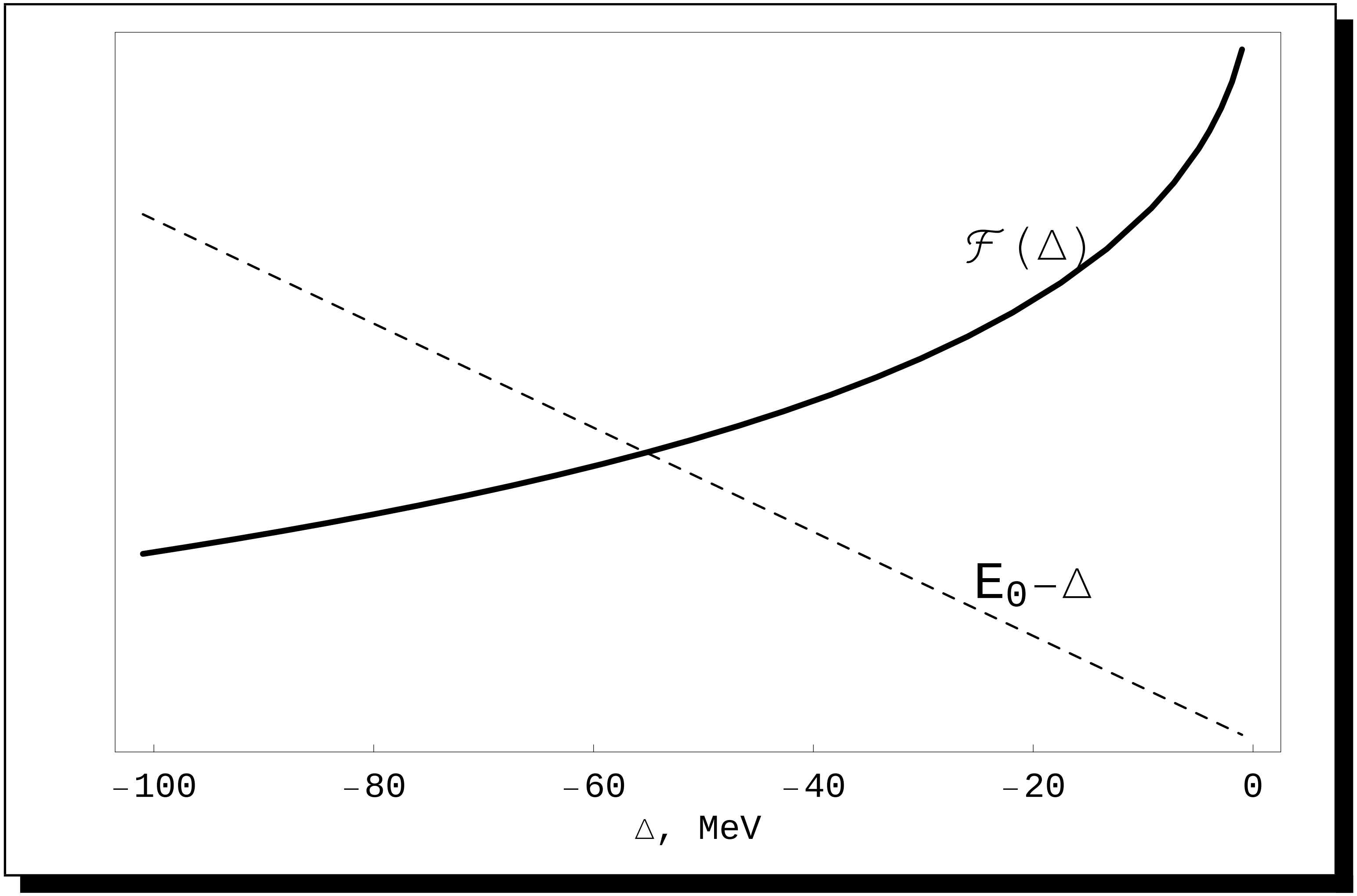
where

$$\boxed{\mathcal{F}(\Delta) \stackrel{\text{def}}{=} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{M_f} \frac{\left|\left\langle M_i \left| \widehat{V} \right| \mathbf{p}, M_f \right\rangle\right|^2}{T_f(\mathbf{p}) - \Delta}}$$

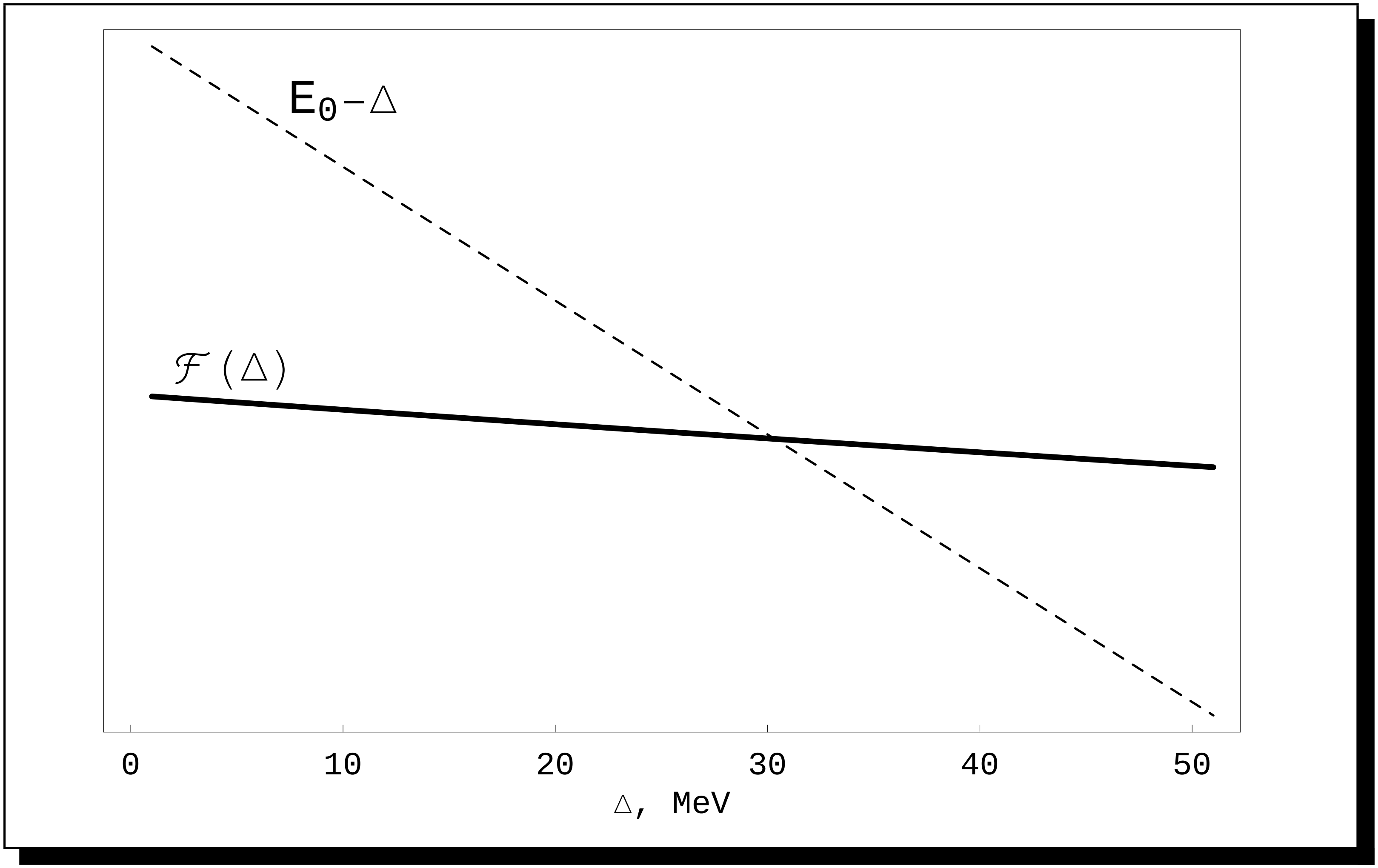
and

$$\boxed{\left\langle M_i \left| \widehat{V} \right| \mathbf{p}, M_f \right\rangle = -\int \Psi^{\dagger}_{D_s}(M_i)\, \sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\, \Psi_D(M_f)\, \frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\, d^3\mathbf{r}}$$

While solving this equation we have two possible situations: $E_0 < E_0^{\text{crit}}$ and $E_0 > E_0^{\text{crit}}$, where $E_0^{\text{crit}} = \mathcal{F}(-0)$.



If $E_0 < E_0^{\text{crit}}$, the equation has a negative real root $\boldsymbol{\Delta} < 0$ and the resulting mass of the D_s meson proves to be under the threshold.



If $E_0 > E_0^{\text{crit}}$, the equation has a complex root $\Delta = \Delta' + i\Delta''$ with positive real part $\Delta' > 0$ and negative imaginary part $\Delta'' < 0$, so the resulting mass of the D_s meson proves to be over the threshold, the meson having a finite width $\Gamma = 2|\Delta''|$. But an analytic continuation of the $\mathcal{F}(\Delta)$ from the upper half-plane is required here.

In D and D_s mesons considered here we treat c-quark as heavy and static so the mesons are described by the following wave functions:

$$\Psi_D\left(J^-, M_f\right) = C^{J,M_f}_{\frac{1}{2},M_f-\frac{1}{2};\frac{1}{2},+\frac{1}{2}} \psi_q^{\frac{1}{2},0,M_f-\frac{1}{2}} \otimes \left|\bar{c}\uparrow\right\rangle + \\ + C^{J,M_f}_{\frac{1}{2},M_f+\frac{1}{2};\frac{1}{2},-\frac{1}{2}} \psi_q^{\frac{1}{2},0,M_f+\frac{1}{2}} \otimes \left|\bar{c}\downarrow\right\rangle$$

$$\Psi_{D_s}\left(J_j^+, M_i\right) = C^{J,M_i}_{j,M_i-\frac{1}{2};\frac{1}{2},+\frac{1}{2}} \psi_s^{j,1,M_i-\frac{1}{2}} \otimes \left|\bar{c}\uparrow\right\rangle + \\ + C^{J,M_i}_{j,M_i+\frac{1}{2};\frac{1}{2},-\frac{1}{2}} \psi_s^{j,1,M_i+\frac{1}{2}} \otimes \left|\bar{c}\downarrow\right\rangle$$

where $\psi^{jlm}_{q,s}$ is the light quark wave function – Dirac bispinor.

For D, D_s mesons with given parity several quantum sets are possible:

D			
J^P	j	l	$\varkappa$
0^-	$\frac{1}{2}$	0	-1
1^-	$\frac{1}{2}$	0	-1
1^-	$\frac{3}{2}$	2	+2

D_s			
J^P	j	l	$\varkappa$
0^+	$\frac{1}{2}$	1	+1
1^+	$\frac{1}{2}$	1	+1
1^+	$\frac{3}{2}$	1	-2
2^+	$\frac{3}{2}$	1	-2
2^+	$\frac{5}{2}$	3	+3

Physical $D_s(1^+)$ states are mixed:

$$\Psi_{D_s}\left(1^+_L, M_i\right) = -\sin\phi\cdot\Psi_{D_s}\left(1^+_{1/2}, M_i\right) + \cos\phi\cdot\Psi_{D_s}\left(1^+_{3/2}, M_i\right),$$

$$\Psi_{D_s}\left(1^+_H, M_i\right) = \cos\phi\cdot\Psi_{D_s}\left(1^+_{1/2}, M_i\right) + \sin\phi\cdot\Psi_{D_s}\left(1^+_{3/2}, M_i\right)$$

Note: we neglect possible mixing between $D(1^-_{1/2})$, $D(1^-_{3/2})$ states and also between $D_s(2^+_{3/2})$, $D_s(2^+_{5/2})$ states.

In what follows short notations for quark bispinors are used:

$$\psi_1(m_1) \stackrel{\text{def}}{=} \psi_s^{\frac{1}{2},1,m_1}, \quad \psi_2(m_2) \stackrel{\text{def}}{=} \psi_q^{\frac{1}{2},0,m_2}, \quad \psi_3(m_3) \stackrel{\text{def}}{=} \psi_s^{\frac{3}{2},1,m_3}$$

The wave function of the light quark q being in the field of the heavy static antiquark $\bar{Q}$ is the solution of the Dirac equation:

$$\boxed{\psi_q^{jlm} = \begin{pmatrix} g(r)\Omega_{jlm} \\ (-1)^{\frac{1+l-l'}{2}} f(r)\Omega_{jl'm} \end{pmatrix}, \quad \int\limits_0^\infty \left(f^2 + g^2\right) r^2 dr = 1}$$

$$\boxed{\begin{aligned} g' + \frac{1+\varkappa}{r} g - (E_q + m_q + U - V) f = 0 \\ f' + \frac{1-\varkappa}{r} f + (E_q - m_q - U - V) g = 0 \end{aligned}}$$

where the interaction between the quark and the antiquark is described by the linear scalar potential and the vector Coulomb potential:

$$\boxed{U = \sigma r, \quad V = -\frac{\beta}{r}, \quad \beta = \frac{4}{3}\alpha_s}$$

Introducing new dimensionless variables

$$\boxed{x = r\sqrt{\sigma}, \quad \varepsilon_q = E_q/\sqrt{\sigma}, \quad \mu_q = m_q/\sqrt{\sigma}}$$

and new dimensionless functions

$$\boxed{g = \sigma^{3/4}\frac{G(x)}{x}, \quad f = \sigma^{3/4}\frac{F(x)}{x}, \quad \int\limits_0^\infty \left(F^2 + G^2\right) dx = 1}$$

we come to the following system of equations

$$\boxed{\begin{aligned} G' + \frac{\varkappa}{x}G - \left(\varepsilon_q + \mu_q + x + \frac{\beta}{x}\right) F = 0 \\ F' - \frac{\varkappa}{x}F + \left(\varepsilon_q - \mu_q - x + \frac{\beta}{x}\right) G = 0 \end{aligned}}$$

which has been analysed further numerically.

We used the following parameters, as usual

$$\sigma = 0.18\ \mathrm{GeV}^2, \quad \alpha_s = 0.39,$$

$$m_s = 210\ \mathrm{MeV}, \quad m_q = 4\ \mathrm{MeV}$$

and obtained Dirac eigenvalues ε

$\varkappa$	$\bar{Q}q,\ \mu_q = 0.01$	$\bar{Q}s,\ \mu_s = 0.5$
-1	1.0026	1.28944
+1	1.7829	2.08607
-2	1.7545	2.08475

and eigenfunctions G, F.

Using explicit expressions for spherical spinors

$$\Omega_{l+1/2,l,m} = \begin{bmatrix} \sqrt{\frac{j+m}{2j}}\, Y_{l,m-1/2} \\ \sqrt{\frac{j-m}{2j}}\, Y_{l,m+1/2} \end{bmatrix}, \quad \Omega_{l-1/2,l,m} = \begin{bmatrix} -\sqrt{\frac{j-m+1}{2j+2}}\, Y_{l,m-1/2} \\ \sqrt{\frac{j+m+1}{2j+2}}\, Y_{l,m+1/2} \end{bmatrix},$$

and expansion of a plane wave on spherical functions

$$e^{i\mathbf{pr}} = 4\pi \sum_{l,m} i^l j_l(pr) Y^*_{l,m}\left(\frac{\mathbf{p}}{p}\right) Y_{l,m}\left(\frac{\mathbf{r}}{r}\right)$$

after long cumbersome transformations . . .

we obtain

$$\left\|\mathcal{V}_{12}\right\|_{m1,m2} = -\int \psi_1^\dagger(m_1)\,\sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\,\psi_2(m_2)\,\frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\,d^3\mathbf{r} =$$

$$= \frac{\sqrt{\sigma}}{f_K\sqrt{\omega_K(p)}}\Phi_0\left(\frac{p}{\sqrt{\sigma}}\right)\sqrt{4\pi}Y^*_{0,m_1-m_2}\left(\frac{\mathbf{p}}{p}\right)$$

$$\left\|\mathcal{V}_{32}\right\|_{m3,m2} = -\int \psi_3^\dagger(m_3)\,\sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\,\psi_2(m_2)\,\frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\,d^3\mathbf{r} =$$

$$= -\frac{\sqrt{\sigma}}{f_K\sqrt{\omega_K(p)}}\Phi_2\left(\frac{p}{\sqrt{\sigma}}\right)\sqrt{\frac{4\pi}{5}}Y^*_{2,m_3-m_2}\left(\frac{\mathbf{p}}{p}\right)\cdot\begin{bmatrix} -1 & +2 \\ -\sqrt{2} & +\sqrt{3} \\ -\sqrt{3} & +\sqrt{2} \\ -2 & +1 \end{bmatrix}$$

where

$$\Phi_0(q) = \int_0^\infty j_0(qx)x dx \Big[G_1(x)F_2(x) - F_1(x)G_2(x)\Big],$$

$$\Phi_2(q) = \int_0^\infty j_2(qx)x dx \Big[G_3(x)F_2(x) - F_3(x)G_2(x)\Big]$$

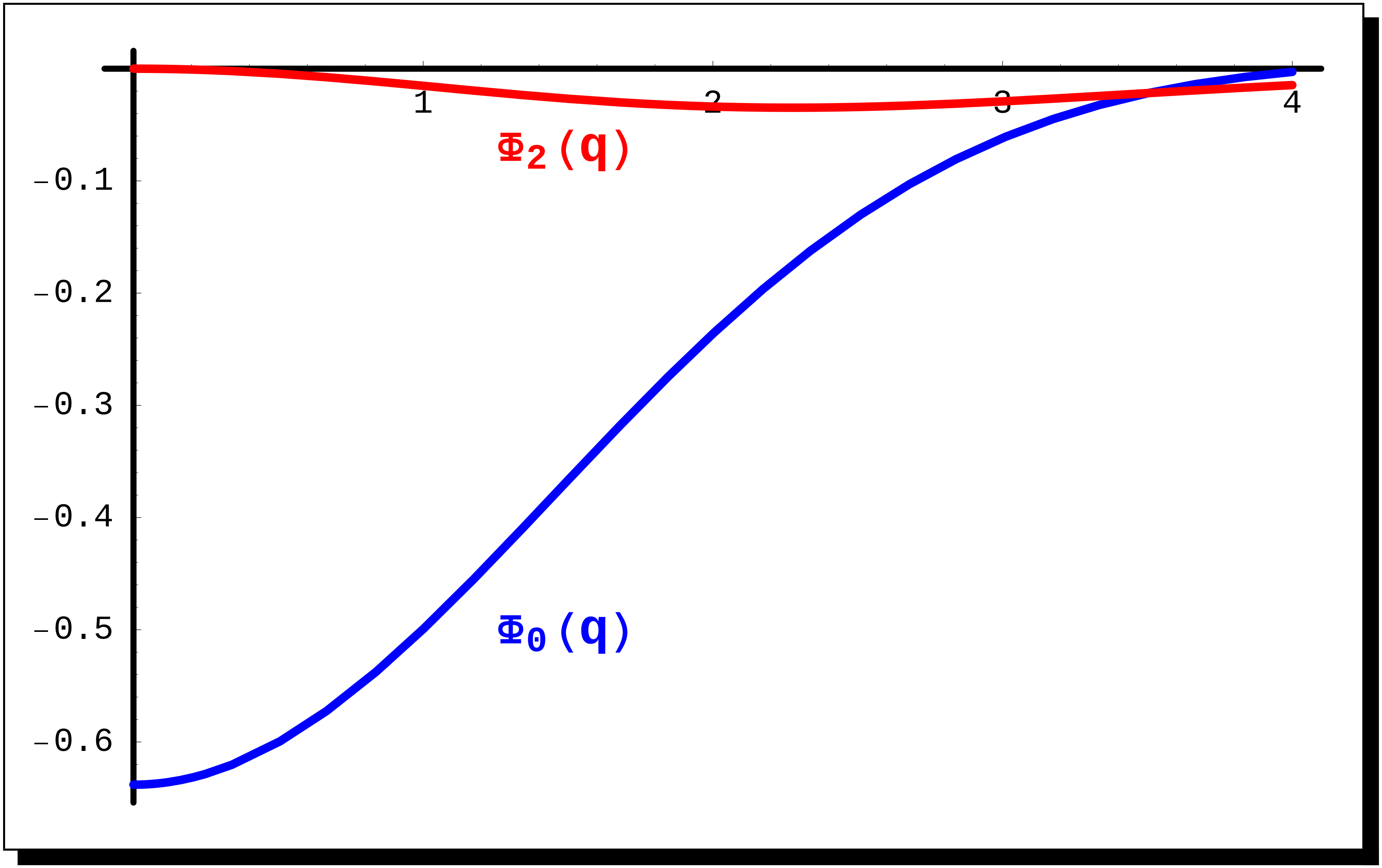
Finally, introducing functions

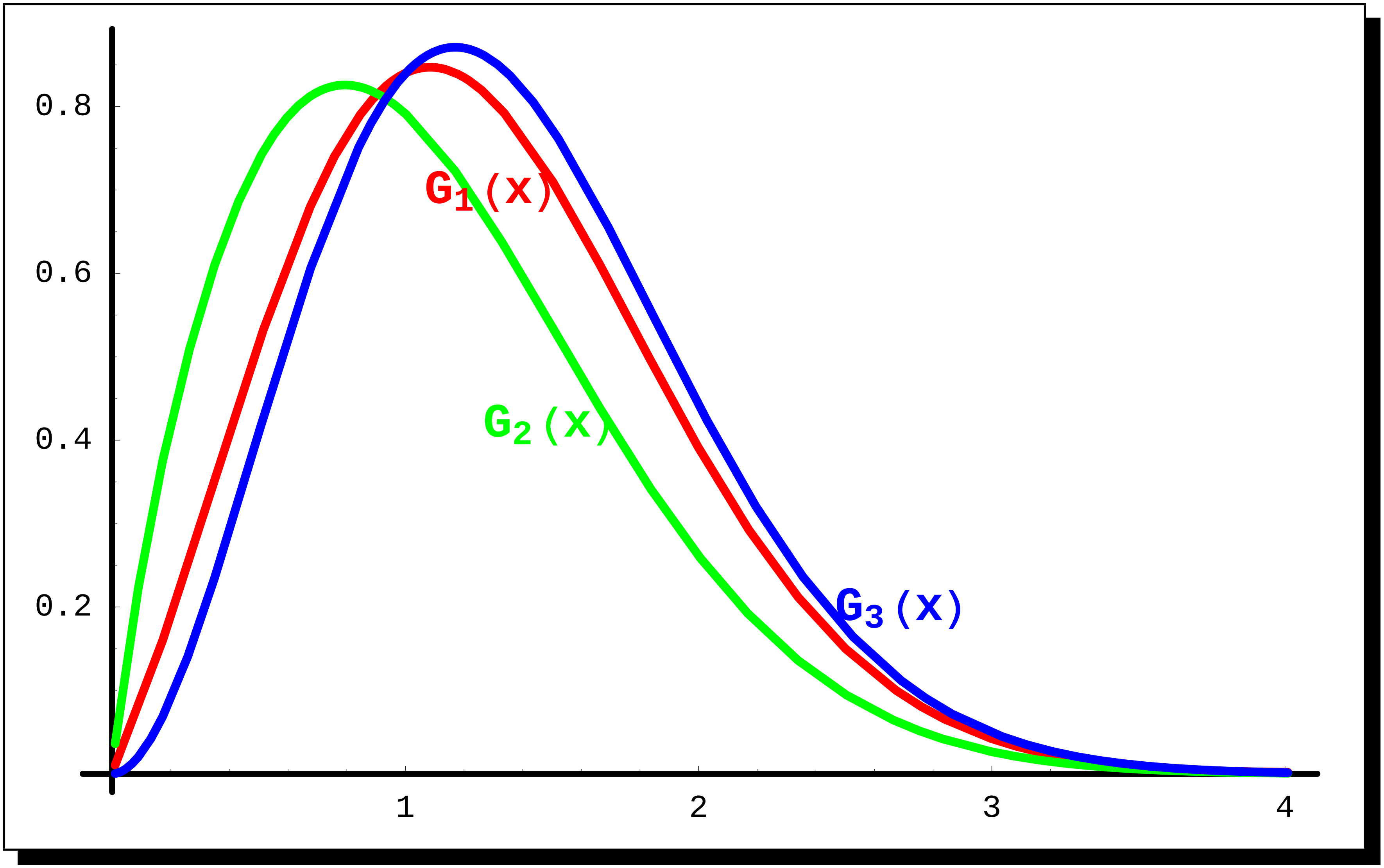
$$\tilde{\mathcal{F}}_{0,2}(\Delta) = \frac{\sigma}{2\pi^2 f_K^2} \int_0^\infty \frac{p(T_f)\omega_D(T_f)dT_f}{T_f + m_D + m_K} \cdot \frac{\Phi_{0,2}^2\left(\frac{p(T_f)}{\sqrt{\sigma}}\right)}{T_f - \Delta}$$

$$\tilde{\Gamma}_{0,2}(T_f) = \frac{\sigma}{\pi f_K^2} \cdot \frac{p(T_f)\omega_D(T_f)}{T_f + m_D + m_K} \cdot \Phi_{0,2}^2\left(\frac{p(T_f)}{\sqrt{\sigma}}\right)$$

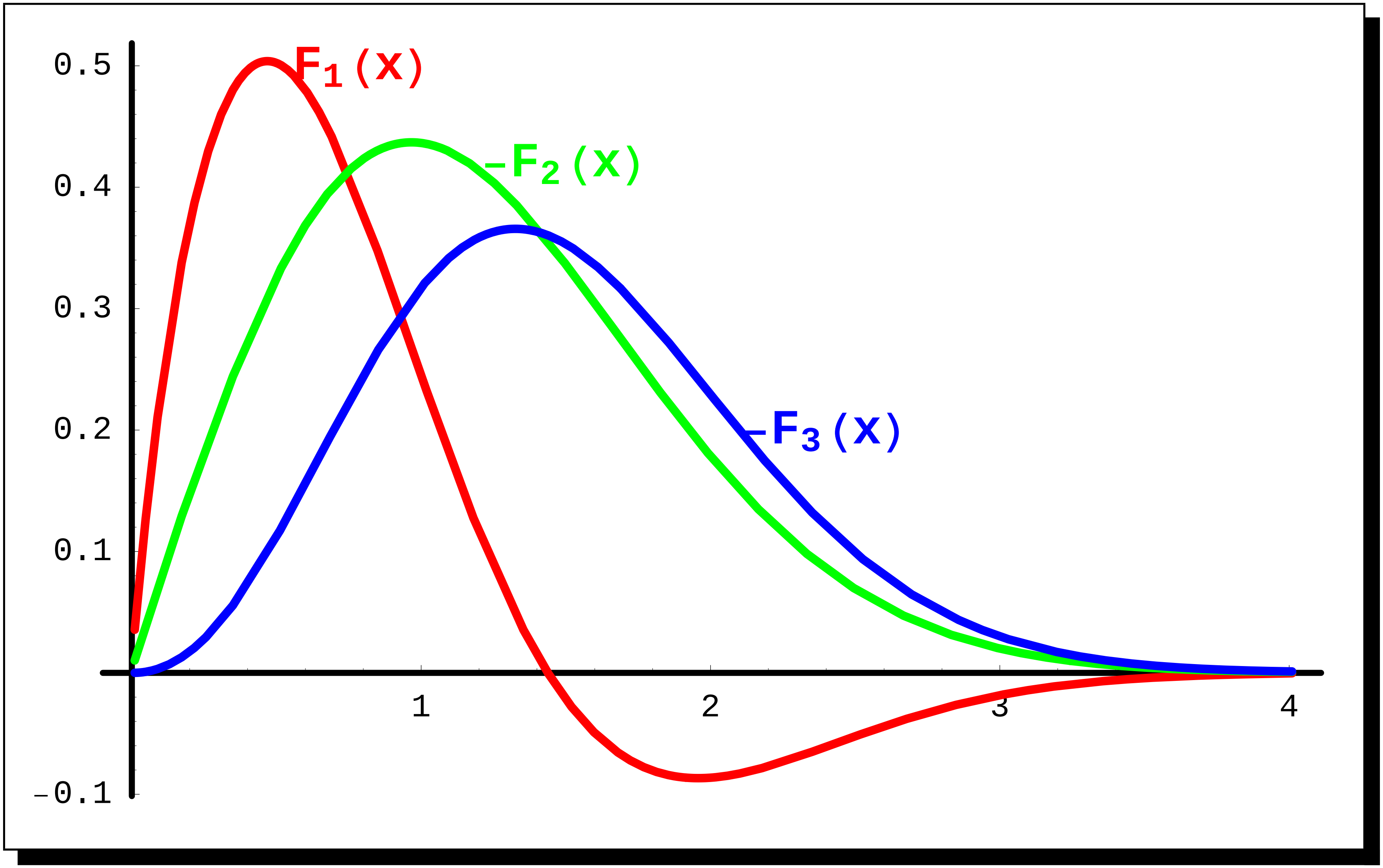
we obtain the following equations to determine meson masses and widths:

$D_s(0^+)$	$E_0[0^+] - \Delta = \tilde{\mathcal{F}}_0(\Delta)$
$D_s(1^+_H)$	$E_0[1^+_H] - \Delta = \cos^2\phi \cdot \tilde{\mathcal{F}}_0(\Delta) + \sin^2\phi \cdot \tilde{\mathcal{F}}_2(\Delta)$
$D_s(1^+_L)$	$E_0[1^+_L] - \Delta' = \sin^2\phi \cdot \tilde{\mathcal{F}}_0(\Delta') + \cos^2\phi \cdot \tilde{\mathcal{F}}_2(\Delta')$
	$\Gamma[1^+_L] = \sin^2\phi \cdot \tilde{\Gamma}_0(\Delta') + \cos^2\phi \cdot \tilde{\Gamma}_2(\Delta')$
$D_s(2^+_{3/2})$	$E_0[2^+_{3/2}] - \Delta' = \frac{3}{5} \cdot \tilde{\mathcal{F}}_2(\Delta')$
	$\Gamma[2^+_{3/2}] = \frac{3}{5} \cdot \tilde{\Gamma}_2(\Delta')$


1
2
3
4
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Φ2(q)
Φ0(q)



0.8
0.6
0.4
0.2
1
2
3
4
$G_1(x)$
$G_2(x)$
$G_3(x)$



0.5
0.4
0.3
0.2
0.1
-0.1
1
2
3
4
F1(x)
-F2(x)
-F3(x)

Results and discussion

$D_s(0^+)$-meson mass shift due to DK channel and $B_s(0^+)$-meson mass shift due to BK channel. Thresholds: $m_D + m_K = 2363$ MeV, $m_B + m_K = 5772$ MeV.

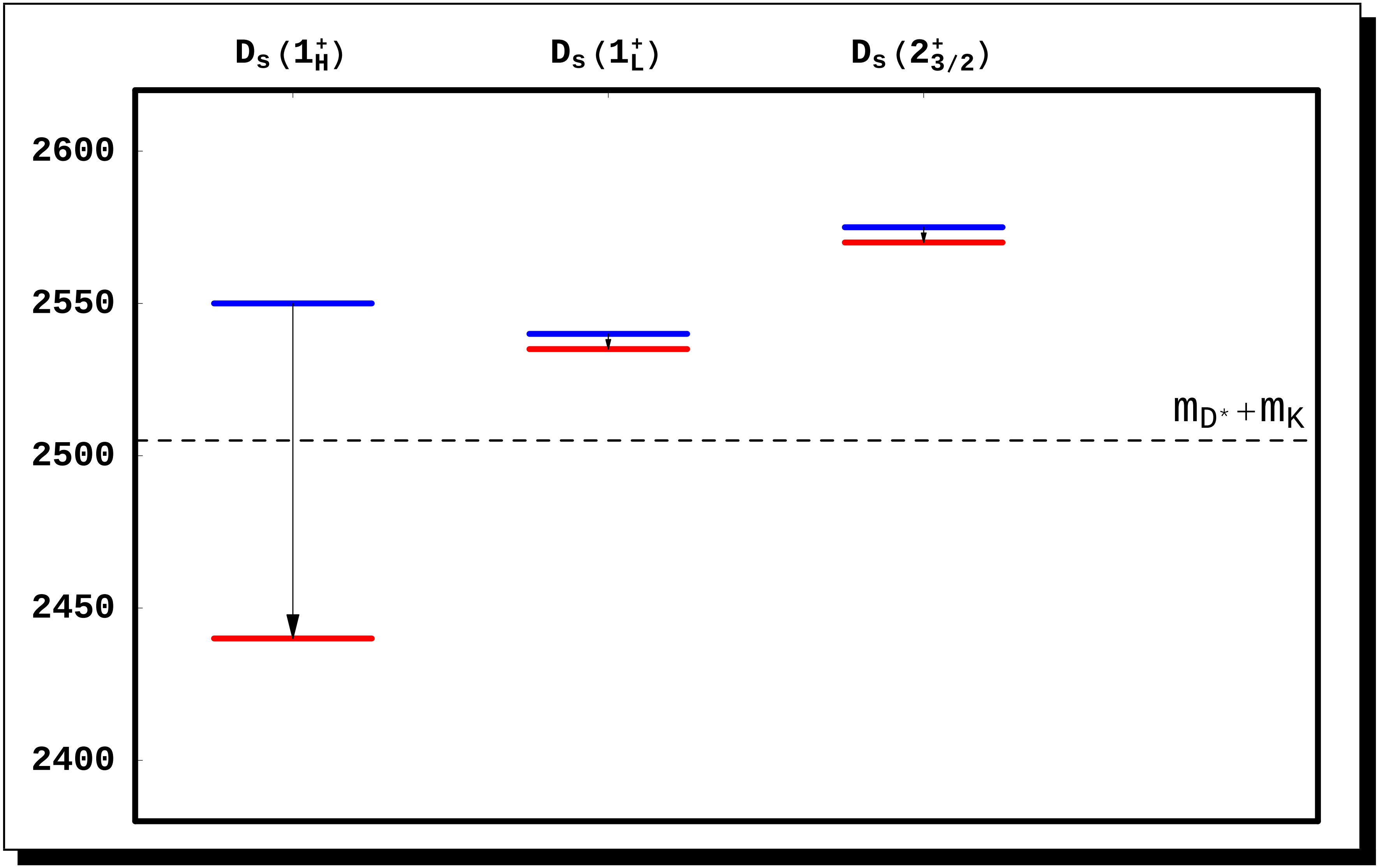
state	$m^{(0)}$	$m^{(\mathrm{theor})}$	$m^{(\mathrm{exp})}$	δm
$D_s(0^+)$	2475(20)	2330(20)	2317	-145
$B_s(0^+)$	5814(15)	5709(15)	not seen	-105

Results on D_s meson mass shift and partial width due to D^*K channel.
Threshold $m_{D^*} + m_K = 2504$ MeV, mixing angle $\sim 4°$.

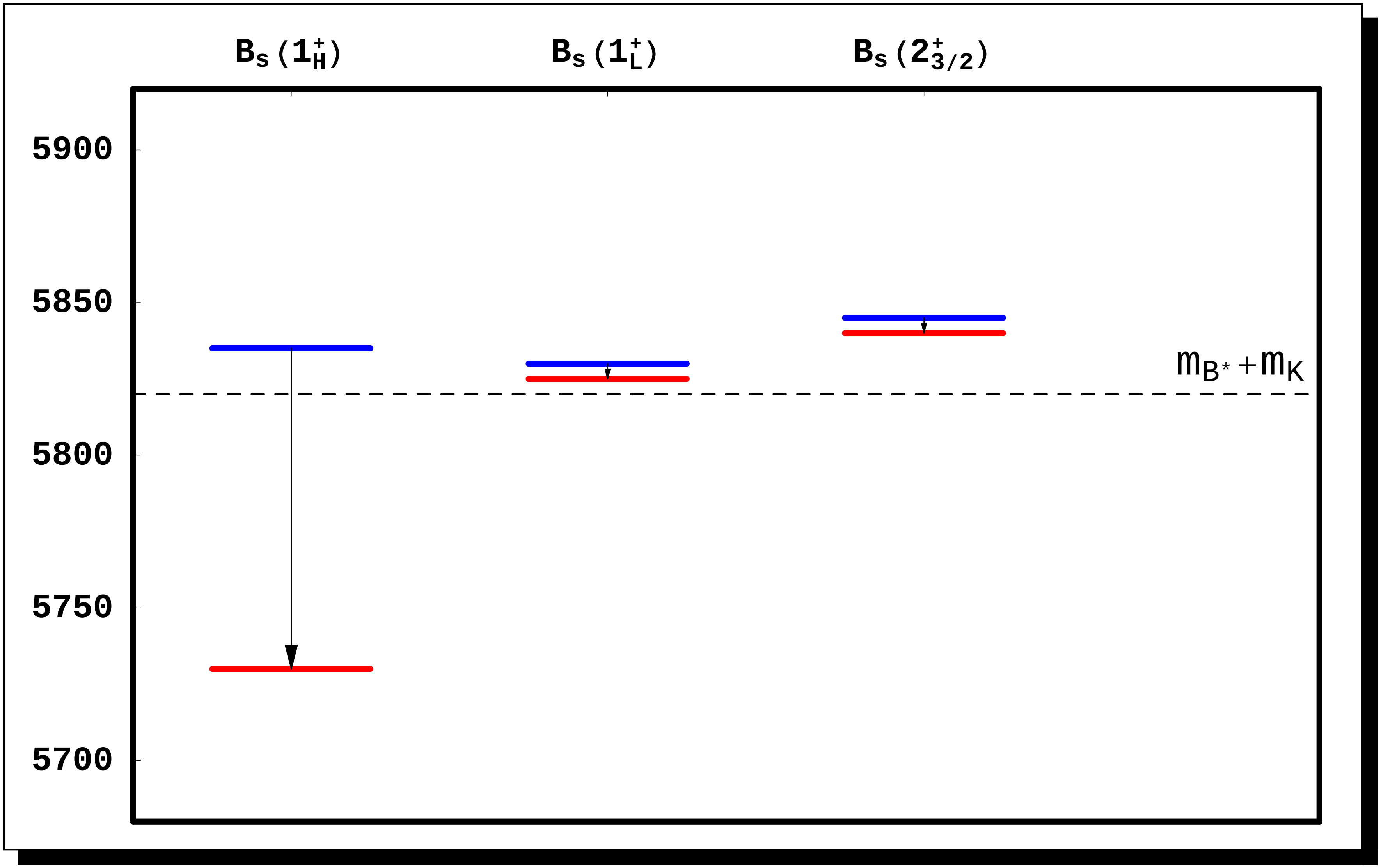
state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm	$\Gamma^{(\text{theor})}_{(D^*K)}$	$\Gamma^{(\text{exp})}_{(D^*K)}$
$D_s(1^+_H)$	2568(15)	2458(15)	2460	-110	$\times$	$\times$
$D_s(1^+_L)$	2537	2535	2535	-2	1.1	< 2.3
$D_s(2^+_{3/2})$	2575	2573	2573	-2	0.03	not seen

Results on B_s meson mass shift and partial width due to B^*K channel.
Threshold $m_{B^*} + m_K = 5819$ MeV, mixing angle $\sim 4°$.

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm	$\Gamma^{(\text{theor})}_{(B^*K)}$	$\Gamma^{(\text{exp})}_{(B^*K)}$
$B_s(1^+_H)$	5835(15)	5727(15)	not seen	-108	$\times$	$\times$
$B_s(1^+_L)$	5830	5828	5829	-2	0.8	< 1.3 ?
$B_s(2^+_{3/2})$	5840	5838	5839	-2	$< 10^{-3}$	not seen



$D_s(1^+_H)$
$D_s(1^+_L)$
$D_s(2^+_{3/2})$
2600
2550
2500
2450
2400
$m_{D^*}+m_K$



$B_s(1^+_H)$
$B_s(1^+_L)$
$B_s(2^+_{3/2})$
5900
5850
5800
5750
5700
$m_{B^*}+m_K$

Summary

- Our method yields final masses and widths of D_s and B_s mesons in good agreement with experimental data (better than 20 MeV). The interaction is deduced from QCD and is almost parameter free.

- The suggested mechanism ensures strong shifts $\gtrsim$ 100 MeV for $j = 1/2$ levels, while $j = 3/2$ levels remain almost *in situ*. Also the mechanism may lead to reordering of the initial levels due to chiral coupling.

- We believe that the chiral coupling might be the universal mechanism also responsible for baryon mass shifts.