# The Pion-Quarkonia interaction

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# Contents

## 1. Introduction. The Quest for new strong interaction.

a) Strong coupling of $Q\bar{Q}$ to $(Q\bar{q})(q\bar{Q})$:

$$\Gamma(\psi(3770) \to D\bar{D}) = 23 \text{ MeV}, \quad \Delta E = 30 \div 40 \text{ MeV}$$

$$p - \text{wave !}$$

$$\Gamma(\psi(4S) \to B\bar{B}) = 20 \text{ MeV}, \quad \Delta E = 20 \text{ MeV}$$

$$p - \text{wave !}$$

b) Strong coupling to pions:

pion-quarkonium resonances: $Z(4430)$ in $\pi^{\pm}\psi(2S)$ with $\Gamma = 45^{+35}_{-18}$ MeV

in $B \to KZ^{\pm}(4430)$

$Z(4051)$ and $Z(4248)$ in $\pi\chi_{c1}$ with $\Gamma = 82$ MeV and $\Gamma = 177$ MeV.

c) Eta transitions strong; however $SU(3)$ violation

$$\psi(2S) \to J/\psi\eta, \quad B = 3.09\%$$

$$\Upsilon(4S) \to \Upsilon(1S)\eta, \quad \Gamma = 4 \ \ \mathrm{keV}$$

d) Systems $J/\psi\pi^+\pi^-, \Upsilon(nS)\pi^+\pi^-$ show up as resonances.

Table 1:

| state | $M$ (MeV) | $\Gamma$ (MeV) | $J^{PC}$ | Decay Modes | Production Modes |
|---|---|---|---|---|---|
| $Y_s(2175)$ | $2175 \pm 8$ | $58 \pm 26$ | $1^{--}$ | $\phi f_0(980)$ | $e^+e^-$ (ISR), $J/\psi$ deca |
| $X(3872)$ | $3871.4 \pm 0.6$ | $< 2.3$ | $1^{++}$ | $\pi^+\pi^- J/\psi$,$\gamma J/\psi$ | $B \to KX(3872)$, $p\bar{p}$ |
| $X(3875)$ | $3875.5 \pm 1.5$ | $3.0^{+2.1}_{-1.7}$ | | $D^0\bar{D}^0\pi^0$ | $B \to KX(3875)$ |
| $Z(3940)$ | $3929 \pm 5$ | $29 \pm 10$ | $2^{++}$ | $D\bar{D}$ | $\gamma\gamma$ |
| $X(3940)$ | $3942 \pm 9$ | $37 \pm 17$ | $J^{P+}$ | $D\bar{D}^*$ | $e^+e^- \to J/\psi X(3940$ |
| $Y(3940)$ | $3943 \pm 17$ | $87 \pm 34$ | $J^{P+}$ | $\omega J/\psi$ | $B \to KY(3940)$ |
| $Y(4008)$ | $4008^{+82}_{-49}$ | $226^{+97}_{-80}$ | $1^{--}$ | $\pi^+\pi^- J/\psi$ | $e^+e^-$(ISR) |
| $X(4160)$ | $4156 \pm 29$ | $139^{+113}_{-65}$ | $J^{P+}$ | $D^*\bar{D}^*$ | $e^+e^- \to J/\psi X(4160$ |
| $Y(4260)$ | $4264 \pm 12$ | $83 \pm 22$ | $1^{--}$ | $\pi^+\pi^- J/\psi$ | $e^+e^-$(ISR ) |
| $Y(4350)$ | $4361 \pm 13$ | $74 \pm 18$ | $1^{--}$ | $\pi^+\pi^- \psi'$ | $e^+e^-$(ISR) |
| $Z(4430)$ | $4433 \pm 5$ | $45^{+35}_{-18}$ | ? | $\pi^\pm \psi'$ | $B \to KZ^\pm(4430)$ |
| $Y(4660)$ | $4664 \pm 12$ | $48 \pm 15$ | $1^{--}$ | $\pi^+\pi^- \psi'$ | $e^+e^-$(ISR) |
| $Y_b$ | $\sim 10,870$ | ? | $1^{--}$ | $\pi^+\pi^- \Upsilon(nS)$ | $e^+e^-$ |

## 2. Microscopic: Quark-pion and decay effective Lagrangians from QCD.

### a) Quark-pion Lagrangian

For light quarks of 3 flavors, $f = u, d, s$ and octet of PS mesons one can derive effective Lagrangian (Yu.S.PRD (2002)).

$$S_{q\pi} = -i \int \bar{\psi}^f(x) M(x) \hat{U}^{fg}(x) \psi^g(x) \tag{1}$$

where $M(x) \to M(0) = M_{br}$ is effective mass operator, and

$$\hat{U} = \exp\left(i\gamma_5 \frac{\lambda^a \varphi_a(x)}{f_\pi}\right);$$

$$\varphi_a \lambda_a = \sqrt{2} \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & k^+ \\ \pi^-, & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & k^0 \\ k^-, & \bar{k}^0, & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \tag{2}$$

One can show, that $M_{br} = M(0) \approx f_\pi = 93$ MeV.

Interaction (30) produces $(q\bar{q}\pi)$ decay vertices.

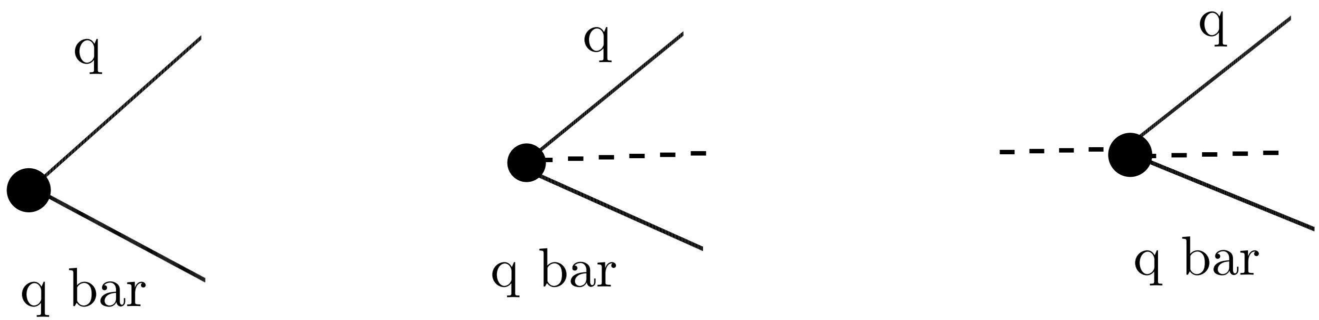
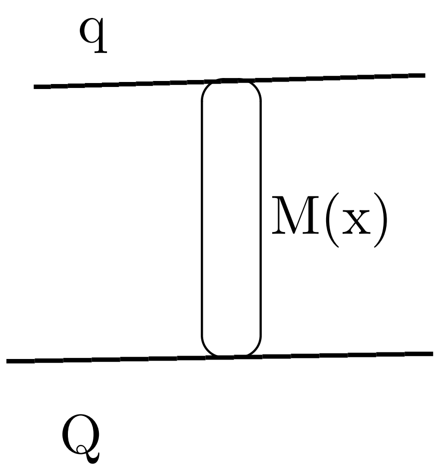



Fig.1

in addition to confinement without pions $(\hat{U} \equiv 1)$

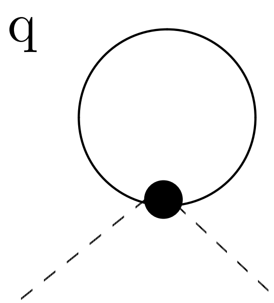



IV

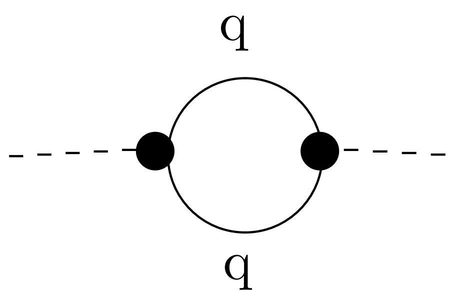
$M(x \to \text{large}) = \sigma|x|$

Using (30) one find $\pi\pi$ vertex

$$W_2(\varphi) = -\frac{N_c}{2} tr(iSM\varphi^2 + SM\gamma_5\varphi S\gamma_5 M\varphi). \tag{3}$$




V




VI

these diagrams cancel when $k_i \to 0$, and $m_\pi \to 0$ (Adler zero)

$$W_2(\varphi) = \frac{1}{2} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \phi_a(k_1) N(k_1, k_2) \phi_a(k_2) \tag{4}$$

$$N(k_1, k_2) = \frac{m_\pi^2}{4N_c} + O(k_1 k_2)$$

and $m_\pi f_\pi^2 = -\frac{m_q}{2N_c} \langle tr \psi \bar{\psi} \rangle$.

One can insert now $W_2(\phi)$ inside heavy quark loop

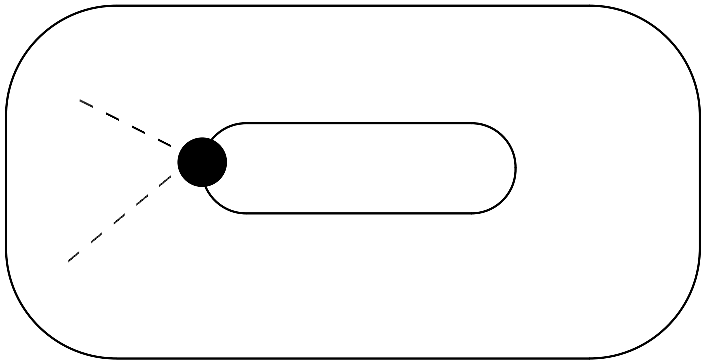


Fig 2 (a)

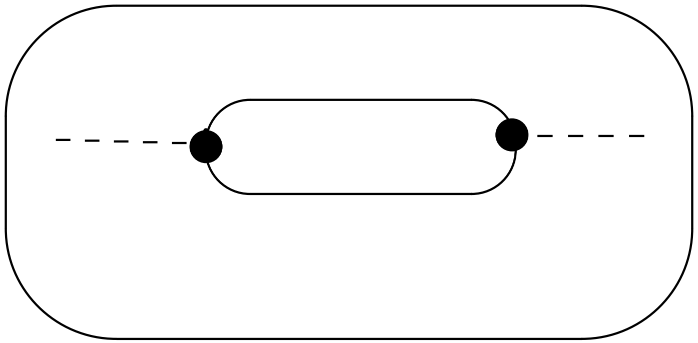


Fig 2 (b)

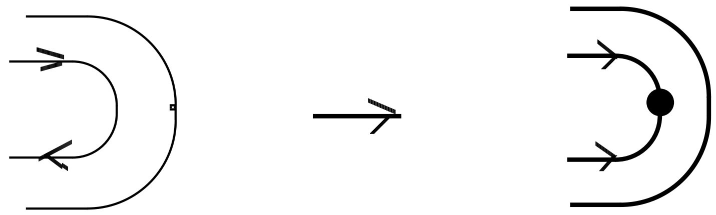
and again require Adler zero.

Hence one has one parameter $M_{br} \approx f_\pi$ for $\pi q\bar{q}$ vertex.

## b) Decay Lagrangian

In Fig.2 a on the left is vertex $M_{br}$. What it is on the r.h.s?

It is the time-turning trajectory (ttt) which is possible only when confinement is working



VII

One can effective replace ttt by the vertex. $\bar{q}M_\omega q$, so that the effective decay Lagrangian

$$L_{ttt} = -i \int d^4x \bar{\psi}(x) M_\omega \psi(x).$$

One can show, that $M_\omega \approx 2\omega \sim 1$ GeV where $\omega$ is average energy of $q$ inside $D$ or $B$ meson.

The total Lagrangian for a light $q\bar{q}$ creation with (or without) pions

$$L_{total} = -i \int d^4x \bar{\psi}(x)(M_{br}e^{i\gamma_5 p} + M_\omega)\psi(x).$$

Note, that $M_\omega \gg M_{br}$, hence we keep only $M_\omega$ when no pions.

## 3. Decay amplitudes *vs* experiment.

Parameter $M_\omega$ is of basic importance: it couples all channels incognito, but shows up in decays.

Also it is large, $M_\omega \approx 1$ GeV.

We must check, how it explains decays

$$\Gamma(\Upsilon(4S) \to B\bar{B})$$

$$\Gamma(\Upsilon(5S) \to B\bar{B}, B\bar{B}^* + c.c., B^*\bar{B}^*, B_s\bar{B}_s, ...)$$

$$\Gamma(\psi(3770) \to D\bar{D}).$$

The width $\Gamma_n$ of the $(nS)$ state is

$$\Gamma_n = \gamma \frac{p_k \tilde{M}_k}{4\pi^2} \int d\Omega_{\mathbf{p}} |J_{nn_2n_3}(\mathbf{p})|^2 \qquad (5)$$

Here $\gamma = \frac{M_\omega^2}{N_c}$ and normalization factor (all traces of Dirac matrices) $y_{123} = \frac{Z_{123x}}{\sqrt{z_1 z_2 z_3}}$.

$k$ - is the decay channel.

$J_{nn_2n_3}(\mathbf{p})$ is the overlap integral

$$J_{n_1n_2n_3}(\mathbf{p}) = \bar{y}_{123} \int d^3(\mathbf{v}-\mathbf{u}) d^3(\mathbf{x}-\mathbf{u}) e^{i\mathbf{pr}} \psi^*_{n_1}(\mathbf{u}-\mathbf{v}) \psi_{n_2}(\mathbf{u}-\mathbf{x}) \psi_{n_3}(\mathbf{x}-\mathbf{v}) \tag{6}$$

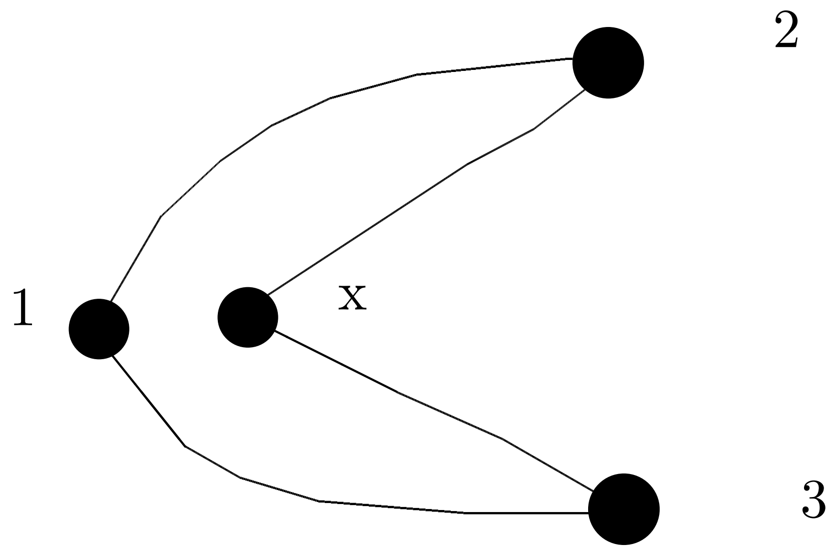
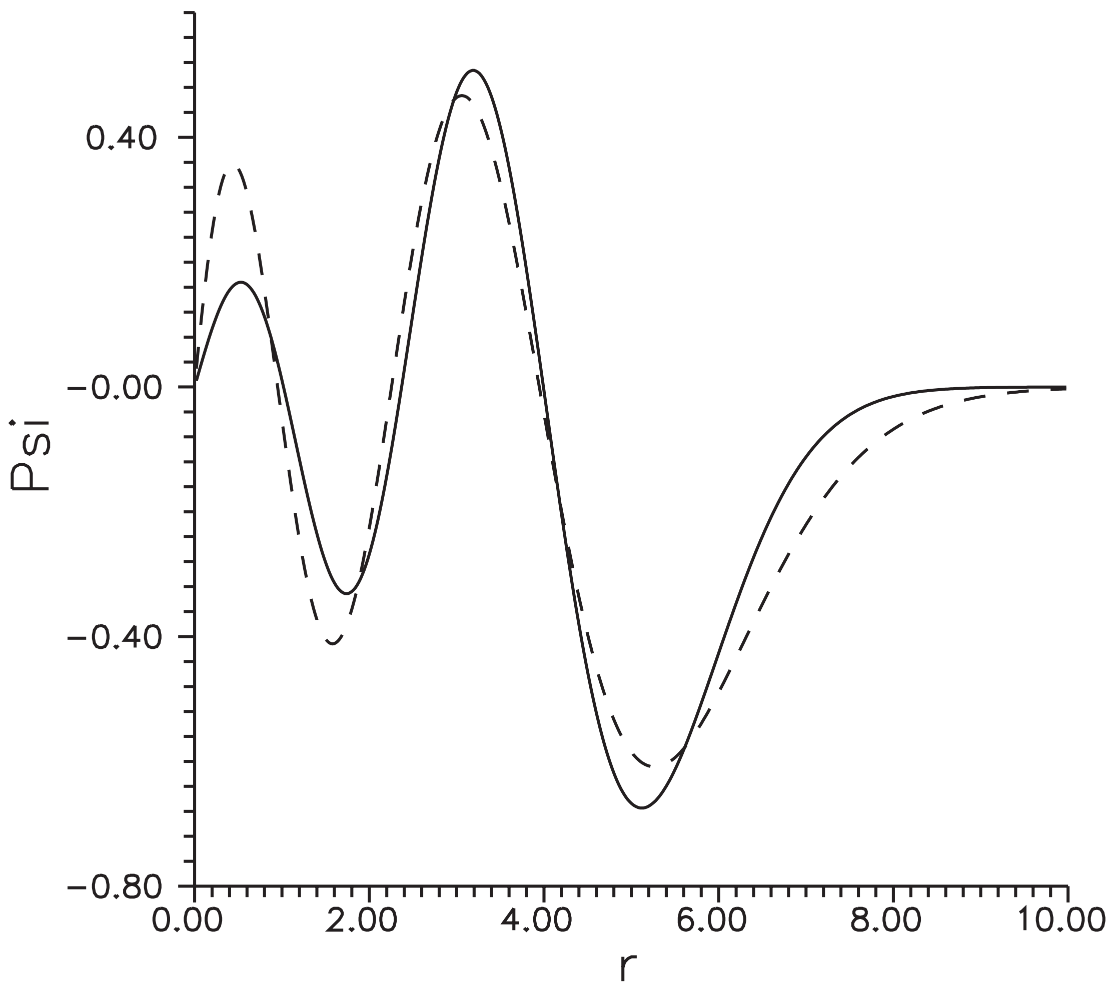
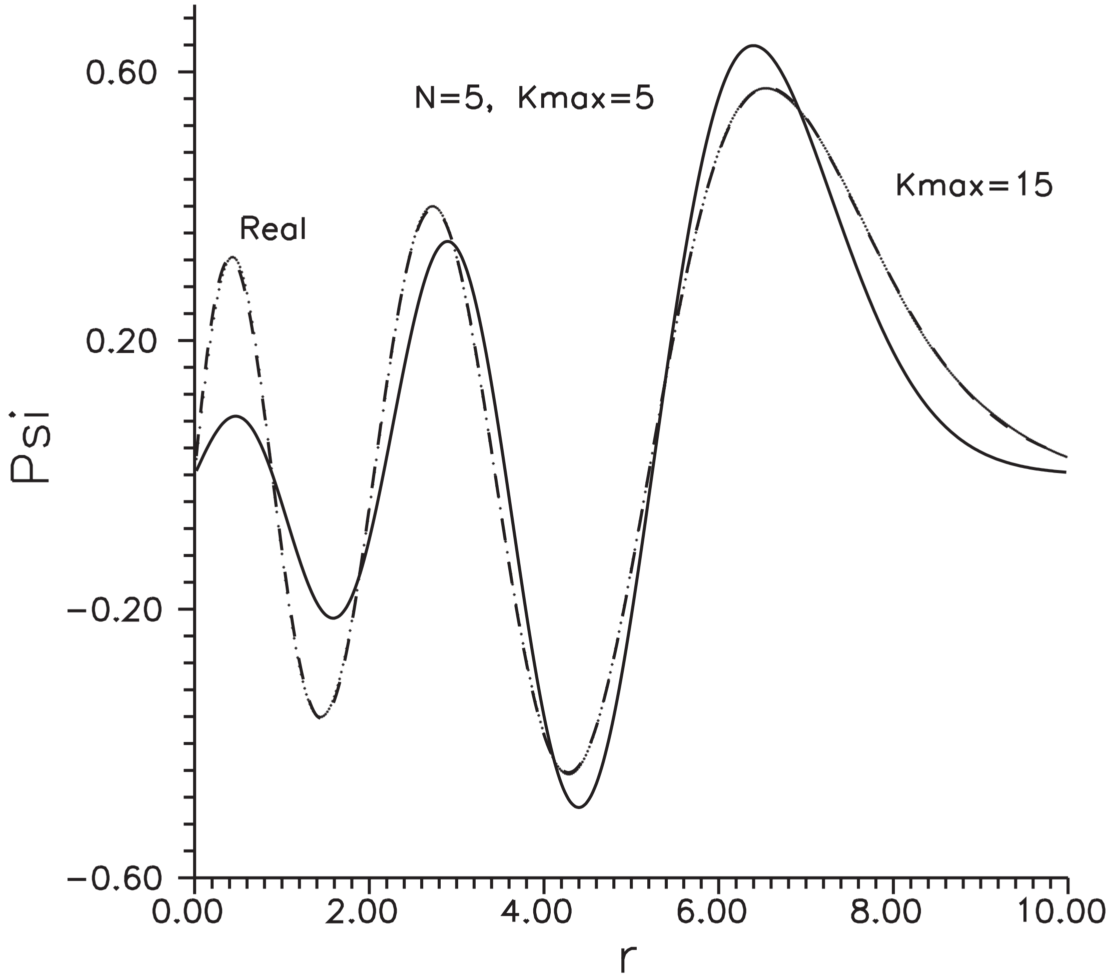


Fig 3

Wave functions are obtained in relativistic string hamiltonian, with all energies of all states within 1-2% from experiment. Sometimes convenient to expand in series of oscillator functions.




Realistic w.f. of $\Upsilon(4S)$ (broken line) and approx. ($k_{\max} = 4$)




Realistic w.f. of $\Upsilon(5S)$ (broken line) $k_{\max} = 15$ (dash-dot), $k_{\max} = 5$ (solid).

Conclusion: choosing $\left(\frac{M_\omega}{2\omega}\right)^2 \approx 0.6$ one has a reasonable agreement with experiment.

| channel | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\Upsilon(5S) \to$ | $BB$ | $BB^*$ | $B^*B^*$ | $B_sB_s$ | $B_sB_s^*$ | $B_s^*B_s^*$ |
| $P_k$,GeV | 1.26 | 1.16 | 1.05 | 0.835 | 0.683 | 0.482 |
| $J_5 = I_5 e^{-\frac{p_k^2}{\Delta}}$ | 0.269 | 0.329 | 0.40 | 0.56 | 0.68 | 0.825 |
| $\left(\frac{2\omega}{M_\omega}\right)^2 \Gamma_k$, MeV | 13.5 | 63.3 | 122 | 8.5 | 27.5 | 25 |

$\bar{\Gamma}_{tot}(5S) = \left(\frac{M_\omega}{2\omega}\right)^2 260$ MeV

Taking $\left(\frac{M_\omega}{2\omega}\right)^2$ from $\Gamma^{th}(4S) = \left(\frac{M_\omega}{2\omega}\right)^2 44$ MeV=$\Gamma^{\exp}(4S) \cong 20$ MeV, one has

$\left(\frac{M_\omega}{2\omega}\right)^2 \approx 1/2$ and $\bar{\Gamma}_{tot}(5S) \approx 130$ MeV.

In experiment $\Gamma_{tot}^{\exp} = (110 \pm 13)$ MeV (CLE3)

## $B$-meson decays of $\Upsilon(5S)$

$\Upsilon(5S) \to$ channel 1,2,3,4,5,6

$BB, BB^*, B^*, B^*, B_sB_s, B_sB_s^*, B_s^*B_s^*$

$$\Gamma_k \equiv \Gamma_{5S}(channel\ k) = \left(\frac{M_\omega}{2\omega}\right)^2 \frac{(p^{(k)})^2 M_k}{6\pi N_c} Z_k^2 |J_{5S}((p^{(k)})|^2$$

coeff. $Z_k^2$ accounts for spin-isospin

$$Z_1^2 = 2Z_4^2 = 1, Z_2^2 = 2Z_5^2 = 4, Z_3^2 = 2Z_6^2 = 7$$

Note, that for $B_s, ...Z_k^2$ are twice as small - no isospin.

Also:

$$\frac{\Gamma_1^{\exp}}{\Gamma_2^{\exp}} < 0.92; \frac{\Gamma_1^{\exp}}{\Gamma_3^{\exp}} < 0.3; \frac{\Gamma_2^{\exp}}{\Gamma_3^{\exp}} = 0.324. \qquad (7)$$

$$\frac{\Gamma_4^{\exp} + \Gamma_5^{\exp} + \Gamma_6^{\exp}}{\Gamma_{tot}} = 0.16 \pm 0.02 \pm 0.058 \qquad (8)$$

To compare with our averaged estimate ($\bar{\Gamma}(B_s)$ is reduced due to decrease of $p^3$)

$$\frac{\bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6}{\Gamma_{tot}} = 0.23$$

Taking into account also also that $\left(\frac{M_\omega}{2\omega}\right)^2 \to \left(\frac{M_\omega}{2\omega_s}\right)^2, \quad \left(\frac{\omega}{\omega_s}\right)^2 = 0.844$. One obtains $\frac{\Gamma_s}{\Gamma_{tot}} = 0.19$. One can see that qualitatively (orders of magnitude) theory is correct, but more work is needed to make precision calculations of $\Upsilon(5S)$.

# Dipion transitions

$$S_{SBr} = -i \int d^\eta x \bar{\psi}^f(x) M_{br} \hat{U}^{fg} \psi^g(x) \tag{9}$$

Pion emission inside $Q\bar{Q}$ system – the $Q\bar{Q}$ Green's function.

$$G_{Q\bar{Q}}^{q\bar{q},\pi\pi}(1;2;E) = \sum_{n,m} \frac{\psi_{Q\bar{Q}}^{(n)}(1) w_{nm}^{(\pi\pi)}(E) \psi_{Q\bar{q}}^{(m)+}(2)}{(E-E_n)(E-E_m)}. \tag{10}$$

where

$$w_{nm}^{(\pi\pi)}(E) = \gamma \left\{ \sum_k \frac{d^3p}{(2\pi)^3} \frac{J_{nn_2n_3}^{(1)}(\mathbf{p},\mathbf{k}_1) J_{mn_2n_3}^{*(1)}(\mathbf{p},\mathbf{k}_2)}{E - E_{n_2n_3}(\mathbf{p}) - E_\pi(\mathbf{k}_1)} + (1 \leftrightarrow 2) \right.$$

$$
- \sum_{n_2' n_3'} \frac{d^3p}{(2\pi)^3} \frac{J^{(2)}_{nn_2'n_3'}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2) J^*_{mn_2'n_3'}(\mathbf{p})}{E - E_{n_2'n_3'}(\mathbf{p}) - E(\mathbf{k}_1,\mathbf{k}_2)} -
$$

$$
- \sum_{k''} \frac{d^3p}{(2\pi)^3} \frac{J_{nn_2''n_3''}(\mathbf{p}) J^{(2)*}_{mn_2''n_3''}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2)}{E - E_{n_2''n_3''}(\mathbf{p})} \Bigg\} \tag{11}
$$

Here $\gamma = \frac{M_{br}^2}{N_c}$; note that $J_{nn_1n_2}(\mathbf{p})$ contains a factor $\left(\frac{M_\omega}{M_{br}}\right)$ due to $q\bar{q}$ creation without pions.

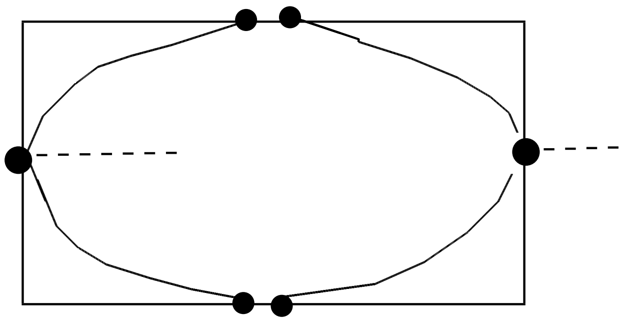


Fig. 4

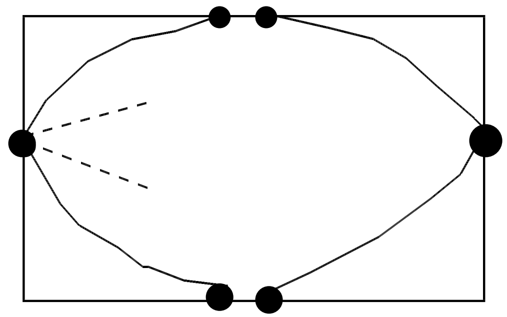


Fig. 5

## Kinematic and phase space

For 3 particles: $\pi_1, \pi_2, \Upsilon(n'S)$ there are 2 standard variables:

$$q^2 \equiv M_{\pi\pi}^2 = (\omega_1 + \omega_2)^2 - \mathbf{K}^2, \quad \mathbf{K} \equiv \mathbf{k}_1 + \mathbf{k}_2$$

$\cos\theta - \theta$ angle of $\pi^+$ w.r.t. to initial momentum.

$M$–initial mass $\Upsilon(nS), M'$- final mass $\Upsilon(n'S), \Delta M = M - M'$

$$\mathbf{k}_1^2 + \mathbf{k}_2^2 = \frac{1}{8M^2}\Big\{ ((M+M')\Delta M + q^2)^2 +$$

$$+(M+M')^2(q^2-4m_\pi^2)\left((\Delta M)^2 - q^2\right)\frac{\cos^2\theta}{q^2}\Big\} - 2m_\pi^2 \equiv \alpha + \gamma\cos^2\theta. \tag{12}$$

$$-\frac{\mathbf{K}^2}{4\beta_2^2} = -\frac{(\Delta M)^2 - q^2}{4\beta_2^2}\left(\frac{(M+M')^2 - q^2}{4M^2}\right) \equiv -\frac{(\Delta M)^2 - q^2}{4\beta_2^2}\cdot(1-\delta). \tag{13}$$

one can write differential rate

$$dw_{nn'}(q, \cos\theta) \equiv d\Phi|\mathcal{M}|^2, \tag{14}$$

where phase space is in $d\Phi$

$$d\Phi = \frac{1}{32\pi^3 N_c^2}\left(\frac{M_{br}}{f_\pi}\right)^4 \frac{(M^2+M'^2-q^2)(M+M')}{4M^3}\times \sqrt{(\Delta M)^2-q^2}\sqrt{q^2-4m_\pi^2}\,d\sqrt{q^2}\,d\cos\theta \tag{15}$$

and matrix element can be written.

Finally matrix element $\mathcal{M}$ can be rewritten as

$$\mathcal{M} = \exp\left(-\frac{\mathbf{k}_1^2+\mathbf{k}_2^2}{4\beta_2^2}\right)\left(\frac{M_{br}}{f_\pi}\right)^2 \mathcal{M}_1 - \exp\left(-\frac{\mathbf{K}^2}{4\beta_2^2}\right)\frac{M_{br}M_\omega}{f_\pi^2}\mathcal{M}_2 \tag{16}$$

## Alder-zero improvement and the final form of $\pi\pi$ spectrum

One can write matrix element as

$$\mathcal{M} = const \left( \bar{a}\ e^{-\frac{\mathbf{k}_1^2+\mathbf{k}_2^2}{4\beta_2^2}} \left( \frac{1}{\frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}^*} + \omega_1 + \Delta M^*_{nn'}} + \frac{1}{\frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}^*} + \omega_2 + \Delta M^*_{nn'}} \right) - \right.$$

$$\left. -\bar{b}\ e^{-\frac{(\mathbf{k}_1+\mathbf{k}_2)^2}{4\beta_2^2}} \left( \frac{1}{\frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}} + \Delta M_{nn'}} + \frac{1}{\frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}} + \Delta M_{nn'} + \omega_1 + \omega_2} \right) \right). \tag{17}$$

Alder zero conditions: for $\mathbf{k}_1 = \omega_1 = 0$ (or $\mathbf{k}_2 = \omega_2 = 0$)$\mathcal{M}$ should vanish. This yields conditions

$$1) \quad \bar{a}(\mathbf{k}_1 = \omega_1 = 0, \quad k_2) = \bar{b}(k_1 = \omega_1 = 0, k_2) \tag{18}$$
$$2) \quad \frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}^*} + \Delta M^*_{nn'} = \frac{\langle \mathbf{p}^2 \rangle}{2\tilde{M}} + \Delta M_{nn'} \equiv \tau_{nn'}.$$

As a result one arrives at the AZI form

$$\mathcal{M}_{nn'} = const \left( e^{-\frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{4\beta_2^2}} \left( \frac{1}{\tau_{nn'}(\omega_1) + \omega_1} + \frac{1}{\tau_{nn'}(\omega_2) + \omega_2} \right) - \right.$$
$$\left. e^{-\frac{(\mathbf{k}_1 + \mathbf{k}_2)^2}{4\beta_2^2}} \left( \frac{1}{\tau_{nn'}(0)} + \frac{1}{\tau_{nn'}(\omega_1 + \omega_2) + \omega_1 + \omega_2} \right) \right). \tag{19}$$

This equation satisfies conditions 1) and 2) for any $\tau_{nn'}(\omega)$. In what follows we perform two calculations

I) without AZI and calculating explicitly all matrix elements

II) with AZI, i.e. using Eq. (52) and finding $\tau_{nn'}(\omega)$ from concrete calculation.

## Simple parametrization

$$x = \frac{q^2 - 4m_\pi^2}{\mu^2}, \quad \mu^2 \equiv (\Delta M)^2 - 4m_\pi^2, \tag{20}$$

In terms of variables $x$, $\cos\theta$ the dipion decay probability can be written as

$$dw_{\pi\pi}(n, n') = C_0 \mu^3 \int_0^1 \sqrt{\frac{x(1-x)}{x + \frac{4m_\pi^2}{\mu^2}}} dx \frac{d\cos\theta}{2} |\mathcal{M}(x, \cos\theta)|^2. \tag{21}$$

Simple form

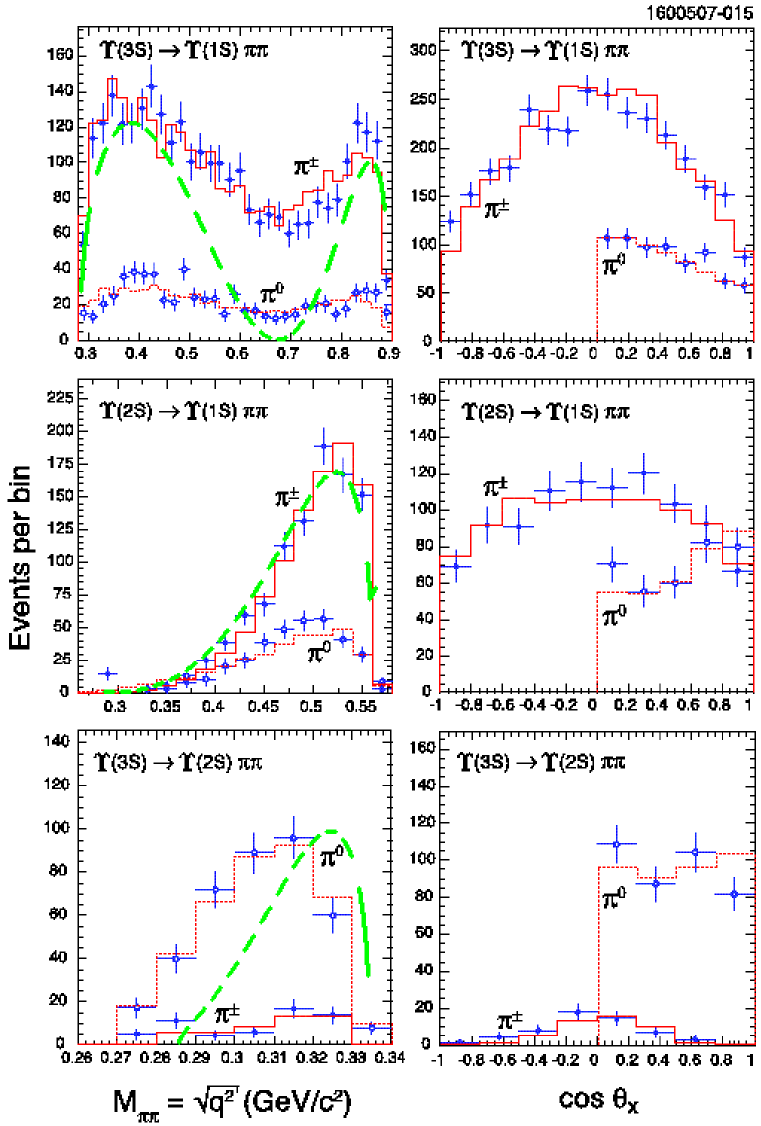
$$\frac{dw}{dq} = const\sqrt{x(1-x)}|\eta(nn') - x|^2 \tag{22}$$

$$const = C_0 \frac{\mu^6}{(4\beta_2^2)^2}|\mathcal{M}_1|^2 \approx 10^{-4}(\mu)^6|\mathcal{M}_1|^2$$

For $\mu \sim O(1)$ GeV, $\mathcal{M}_1 \sim O(1 \text{ GeV}^{-1})$, $\;\Gamma_{\pi\pi} = \int dw = O(10^{-6} \text{ GeV})$

$\Gamma_{\pi\pi} \approx O(1KeV)$

in accordance with experiment.


1600507-015
Υ(3S) → Υ(1S) ππ
Υ(2S) → Υ(1S) ππ
Υ(3S) → Υ(2S) ππ
π±
π0
Events per bin
M_ππ = √q² (GeV/c²)
cos θ_X

We calculate 3 types of theoretical output

1. the di-pion spectrum

$$\frac{dw_{\pi\pi}(n,n')}{dq} = C_0\mu^2\sqrt{x(1-x)}\int_{-1}^{+1} d\cos\theta|\mathcal{M}(x,cos\theta)|^2$$

$$C_0 = \frac{1}{32\pi^3 N_c^2} = 1.12\cdot 10^{-4}$$

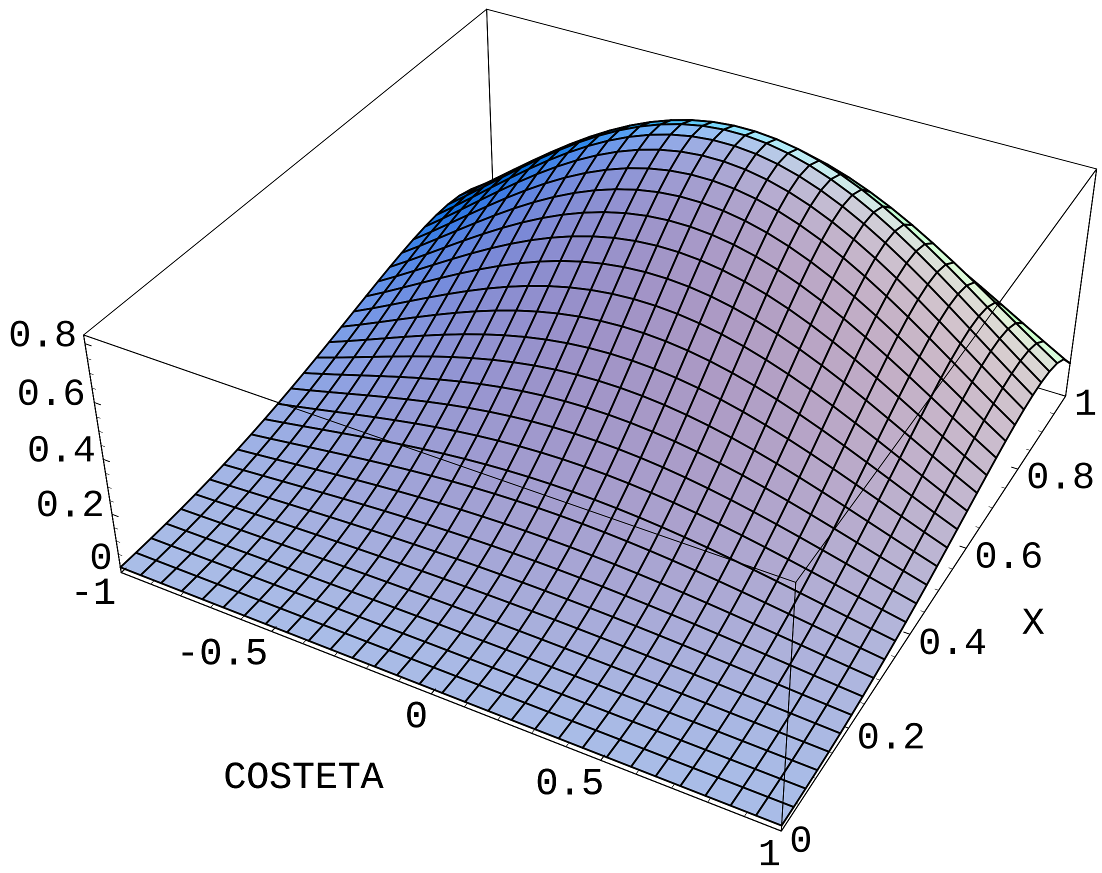
2. the $\cos\theta$ distribution

$$\frac{dw_{\pi\pi}(n,n')}{d\cos\theta} = C_0\mu^3\int_0^1 \sqrt{\frac{x(1-x)}{x+\frac{4m^2}{\mu^2}}}\frac{dx}{2}|\mathcal{M}(x,cos\theta)|^2$$

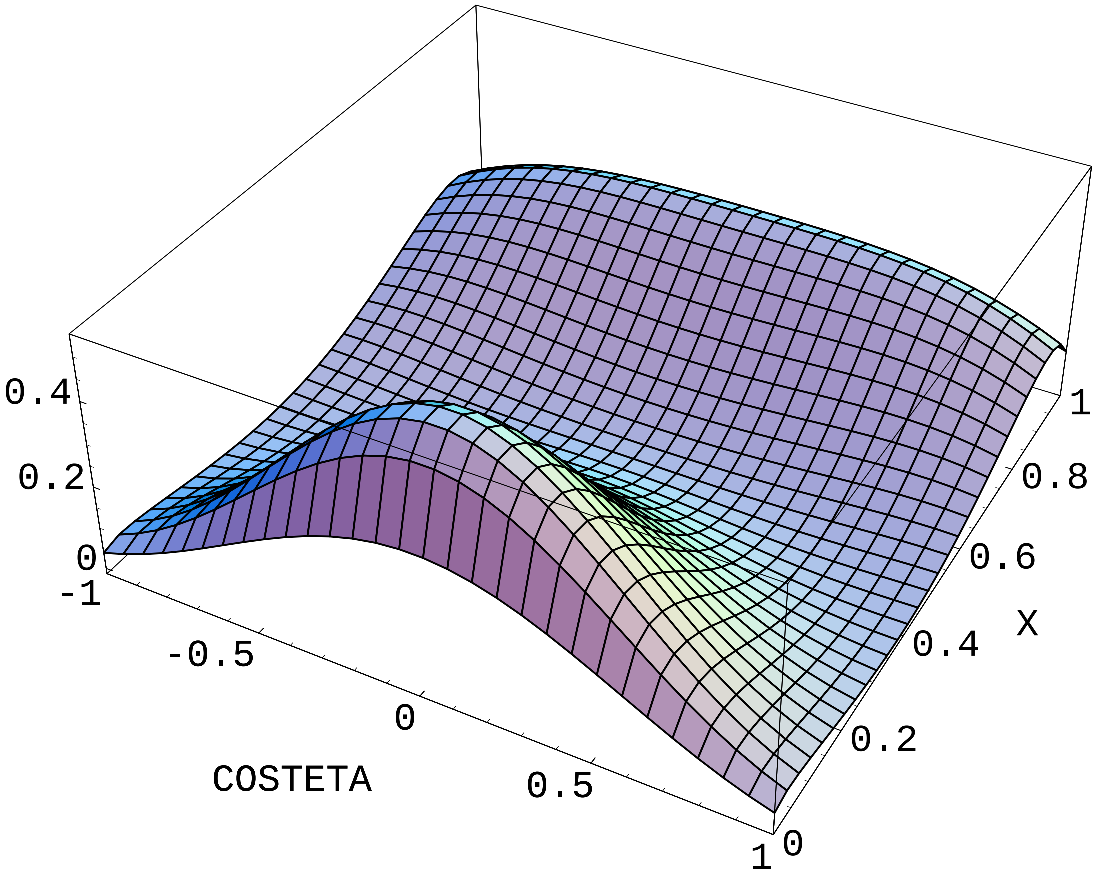
3. the 2d distributions

$$\frac{dw_{\pi\pi}(n,n')}{dxd\cos\theta} = f(x,\cos\theta)$$

For $(n,n') = (2,1),(,1),(3,2),(4,1)$ and (4,2).




(2,1)

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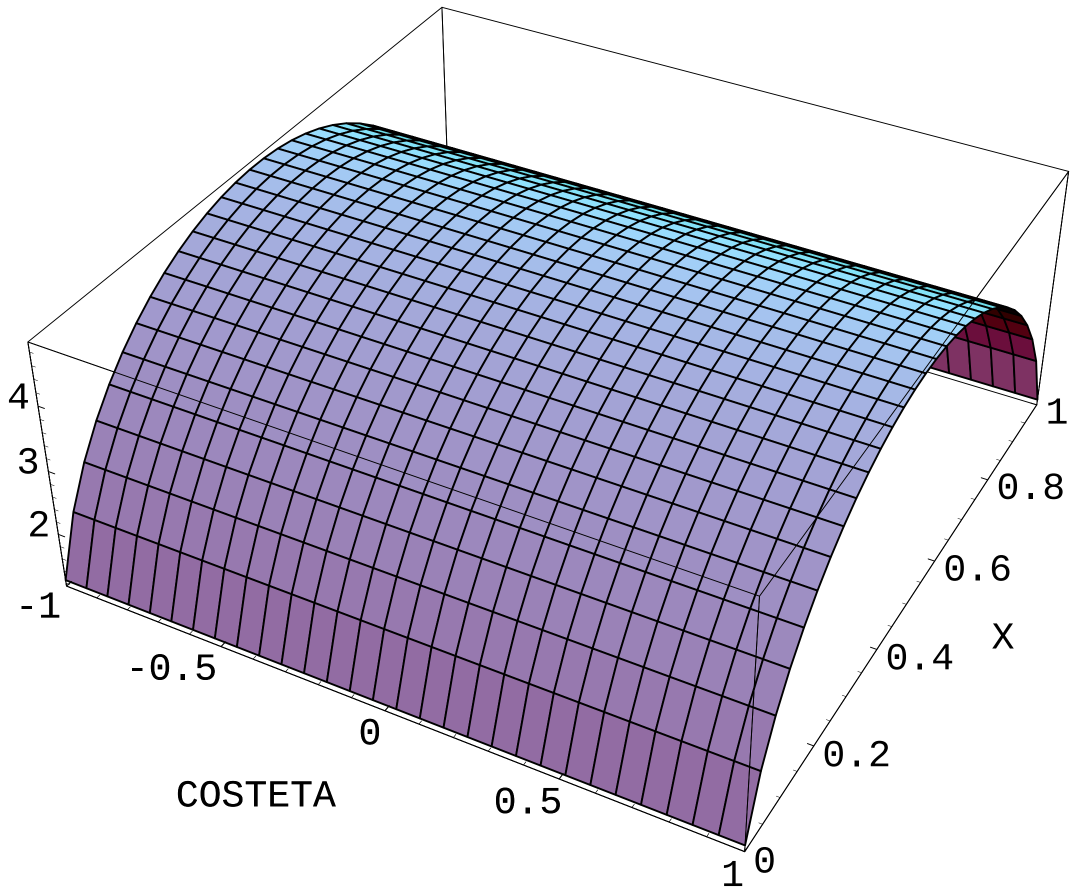
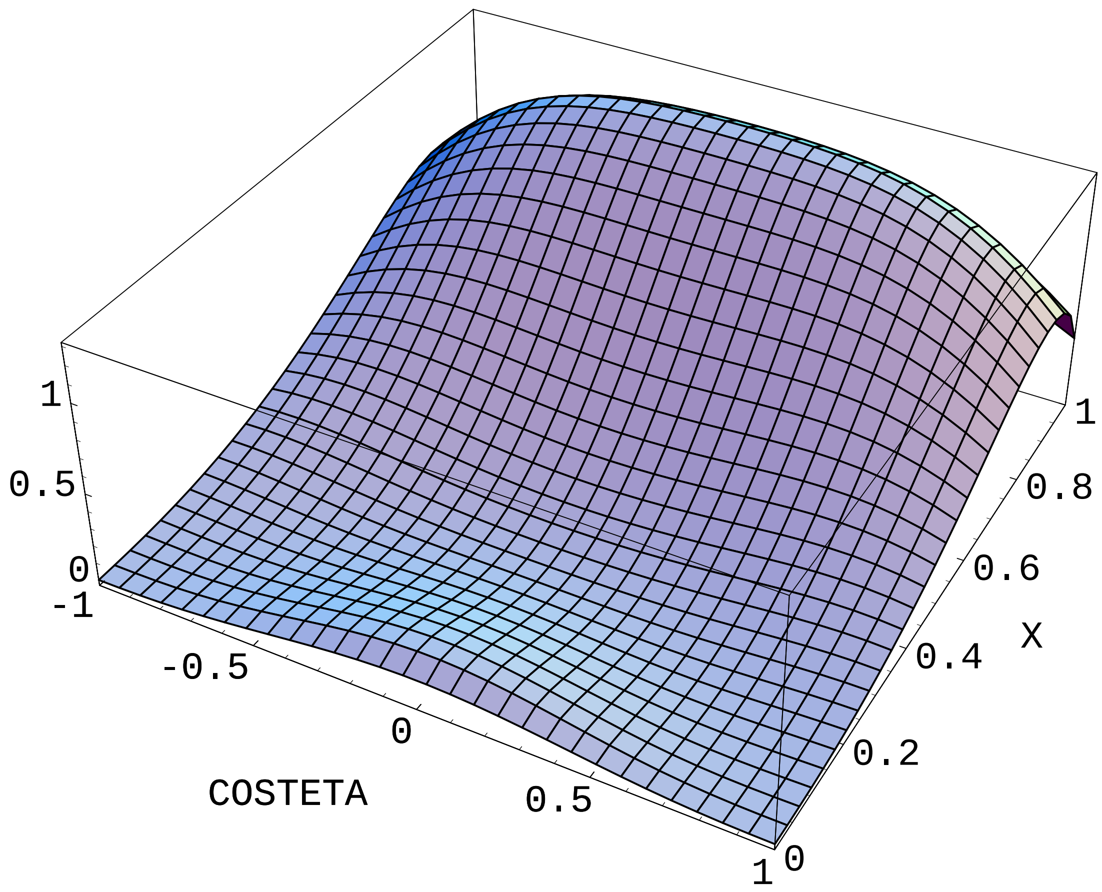
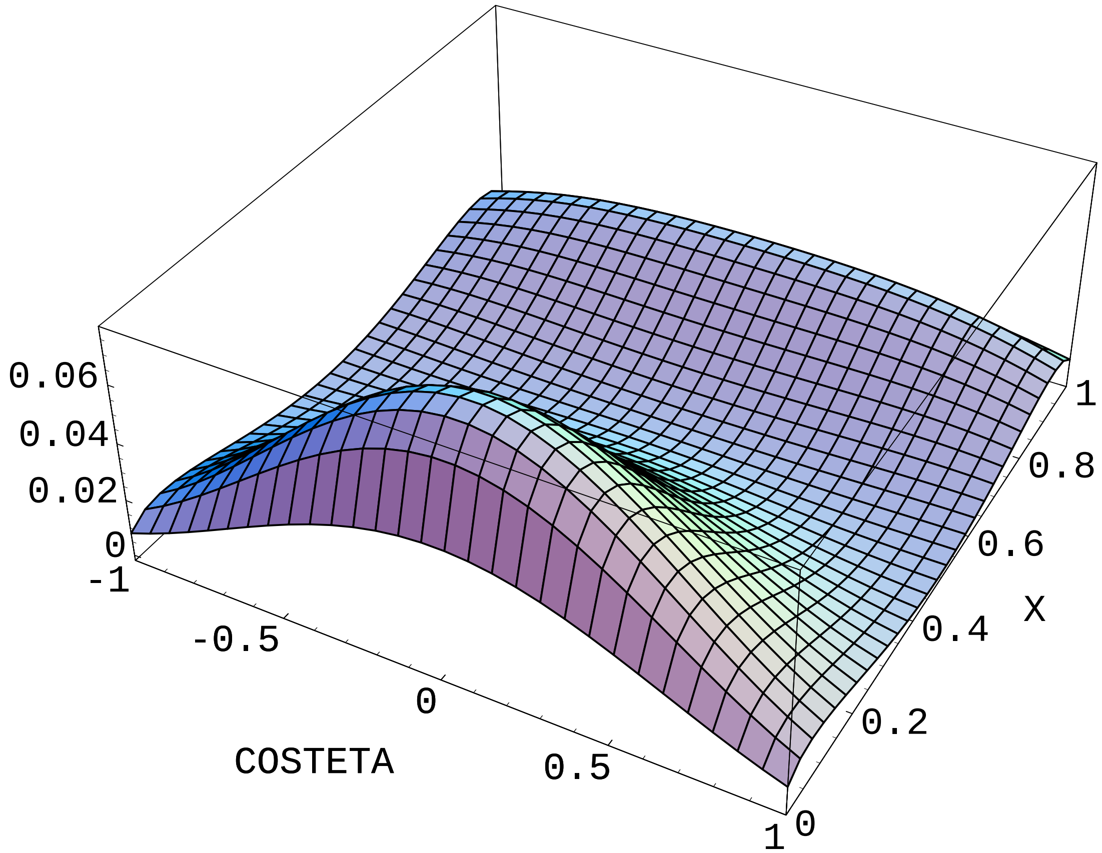



(3,2)




(4,1)




(4,2)

A special check: $\Upsilon(5S) \to \Upsilon(n'S)\pi\pi \; n' = 1, 2, 3.$

New features:

1) The dipion widths $\Gamma_{\pi\pi}(5,1), \Gamma_{\pi\pi}(5,2), \Gamma_{\pi\pi}(5,3)$ are $\sim 1000$ times larger than the corresponding widths for $\Gamma_{\pi\pi}(nn')$ with $n = 2, 3, 4$.

2) The hierarchy of the widths
$\Gamma_{BB}(5S) < \Gamma_{BB^*}(5S) < \Gamma_{B^*B^*}(5S)$occurs in experiment with $\Gamma_{tot}(5S) \sim 0(100 \text{ MeV})$.

3) Dikaon width of $\Upsilon(5S)$ is $\sim 1/10$ of the dipion width.

4) The dipion spectra in (5,1), (5,2) transitions are not similar to spectra found for $n = 2, 3, 4$, showing a possible role of $\pi\pi$ FSI.

$$\mathcal{M} = \exp\left(-\frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{4\beta_2^2}\right)\left(\frac{M_{br}}{f_\pi}\right)^2 \mathcal{M}_1 - \exp\left(-\frac{\mathbf{K}^2}{4\beta_2^2}\right)\frac{M_{br}M_\omega}{f_\pi^2}\mathcal{M}_2. \tag{23}$$

$$\frac{dw_{\pi\pi}(n,n')}{dq d\cos\theta} = C_0 \mu^2 \sqrt{x(1-x)} |\mathcal{M}|^2. \tag{24}$$

Using Adler Zero Requirement (AZR) one can write

$$\mathcal{M} = \bar{\mathcal{M}}(\exp_1 - \exp_2 f(q)), \tag{25}$$

$$f(q) = \alpha f_\sigma(q) + \beta f_{f_0}(q) \tag{26}$$

$$f_\sigma(q) = \left[\frac{m_\sigma^2 - m_\pi^2)^2 + \gamma_\sigma^2}{(m_\sigma^2 - q^2)^2 + \gamma_\sigma^2}\right]^{1/2}, \;\; f_{f_0}(q) = \left[\frac{(m_{f_0}^2 - m_\pi^2)^2 + \gamma_{f_0}^2}{(m_{f_0}^2 - q^2)^2 + \gamma_{f_0}^2}\right]^{1/2} sign(m_{f_0} - q). \tag{27}$$

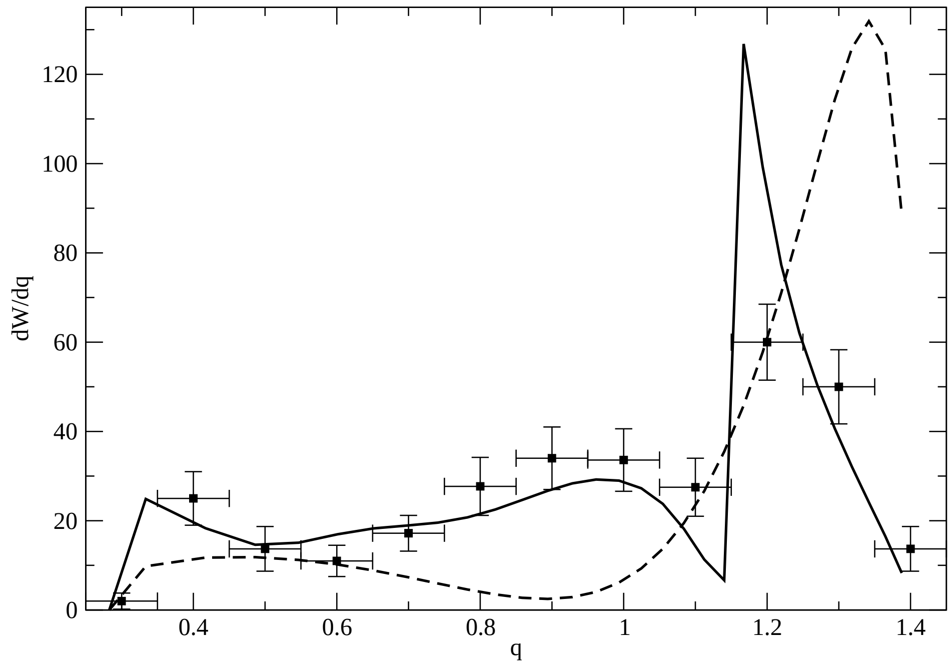



Fig. Comparison of theoretical predictions, Eqs. (25), (26) with experiment [?] for the dipion spectrum, $\frac{dw}{dq}$, in the $\Upsilon(5,1)\pi\pi$ transition. Theory: Eq. (25) with $f \equiv 1$ – broken curve, Eq. (25) with $f$ as in Eq. (26) (parameters given in the text) – solid line. Theoretical curve is normalized to the total experimental width $\Gamma^{\exp}_{\pi\pi} = \frac{dw}{dq} dq$.

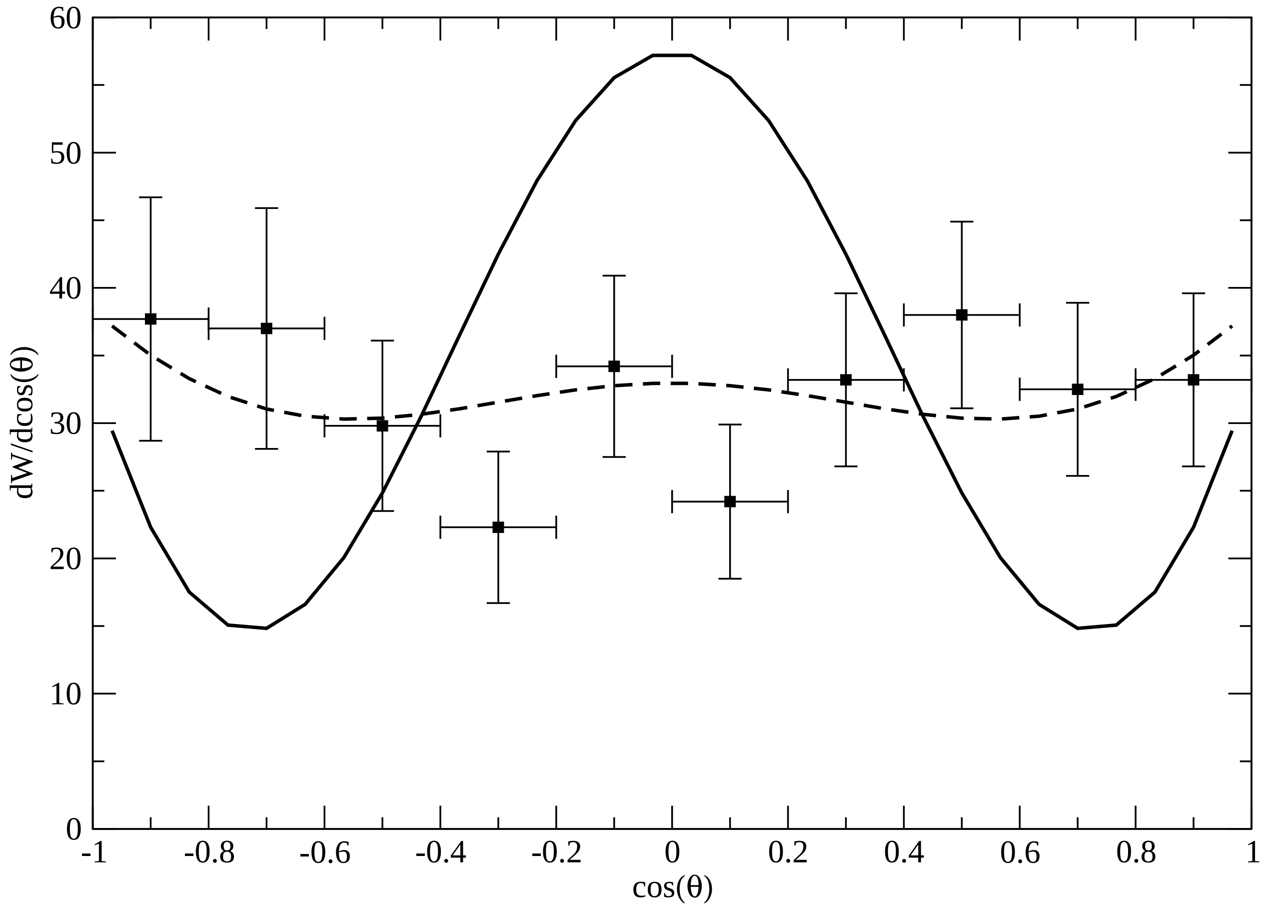



Fig. The same as in Fig.3, for the angular distribution $\frac{dw}{d\cos\theta}$ in the $\Upsilon(5,1)\pi\pi$ transition.

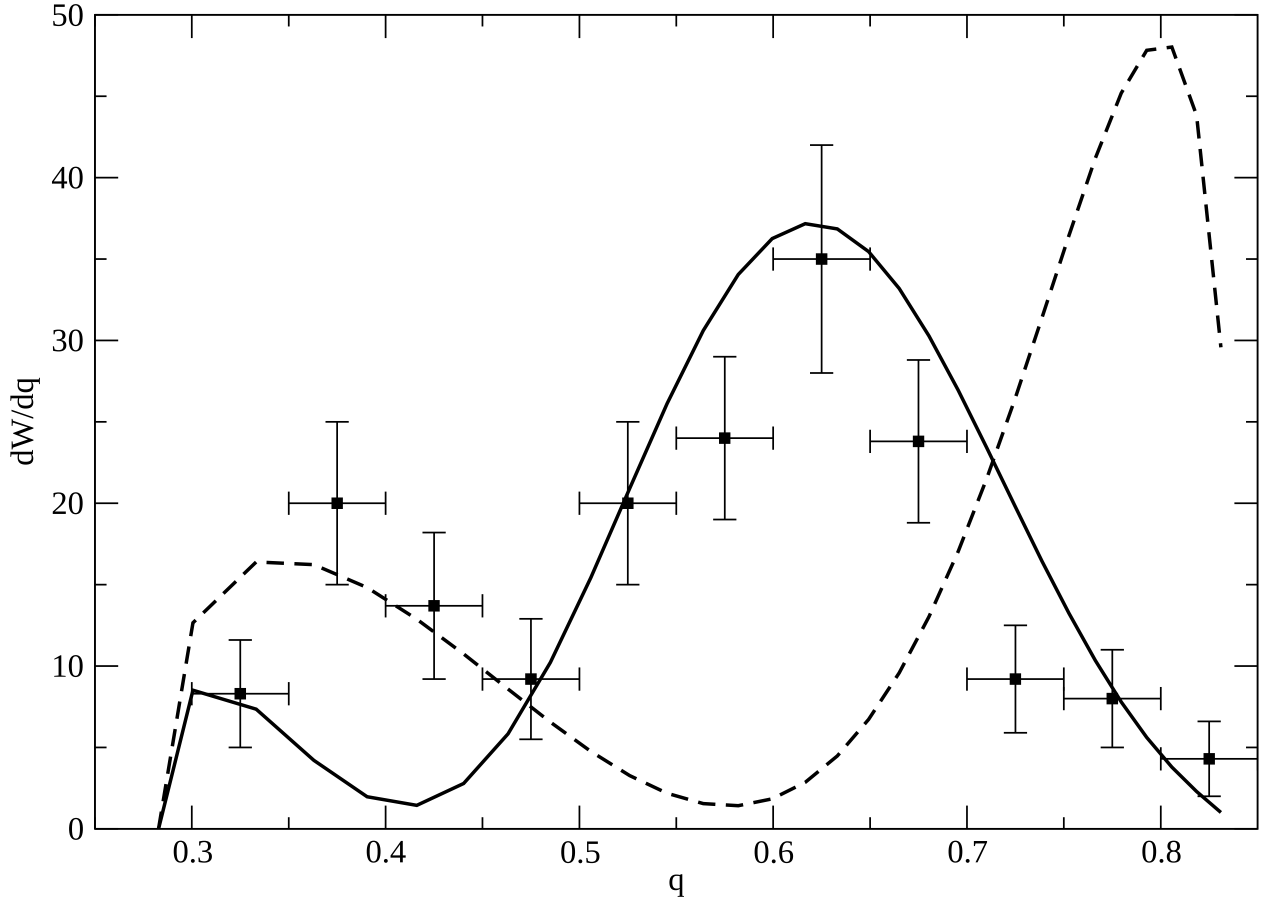



Fig. The same as in Fig.3, for the dipion spectrum $\frac{dw}{dq}$ in the $\Upsilon(5,2)\pi\pi$ transition.

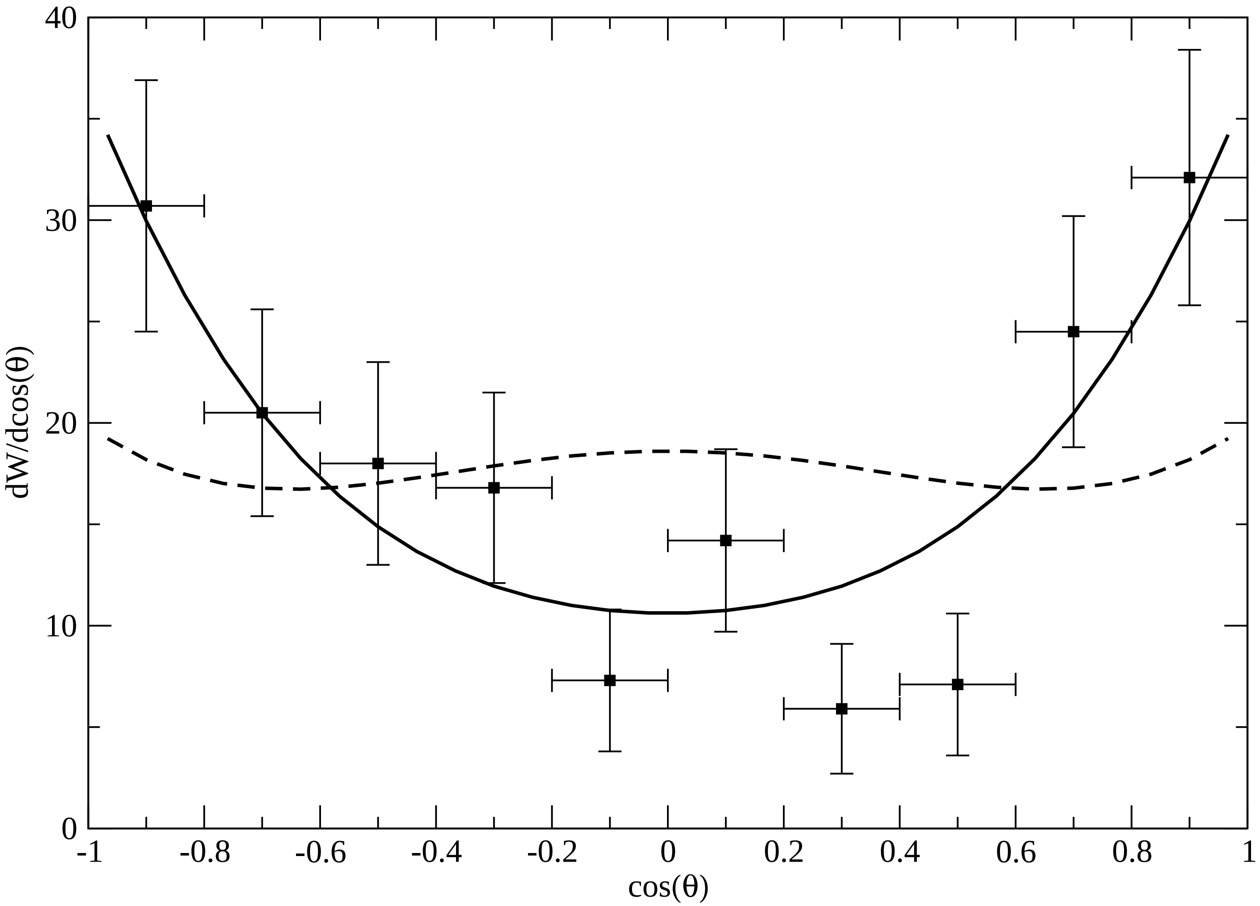



Fig. The same as in Fig.3, for the angular distribution $\frac{dw}{d\cos\theta}$ in the $\Upsilon(5,2)\pi\pi$ transition.

## $K\bar{K}$ transition

$$\Gamma_{KK}(5,1) = C_0 \mu_K^3 \int_0^1 dx \sqrt{\frac{x(1-x)}{x + \frac{4m_K^2}{\mu_K^2}}} \int_{-1}^{+1} \frac{d\cos\theta}{2} |\mathcal{M}_k|^2. \tag{28}$$

Here $\mu_K^2 = (\Delta E)^2 - 4m_K^2 = 0.985$ GeV$^2$, $\mu_K = 0.992$ GeV.

As a result, approximating the ratio of integrals over $dx$ as $1/2$, one obtains

$$\frac{\Gamma_{KK}(5,1)}{\Gamma_{\pi\pi}(5,1)} = \frac{1}{2}\left(\frac{\mu_K}{\mu}\right)^3 \left(\frac{f_\pi}{f_k}\right)^4 = 0.194 \left(\frac{f_\pi}{f_K}\right)^4 = 0.092 \approx 1/10, \tag{29}$$

where we have used $f_\pi = 93$ MeV, $f_K = 112$ MeV [?].

## Single eta emission and breakdown of multipole expansion

Consider

$$\Upsilon(nS) \to \Upsilon(n'S)\eta$$

denoted $\Upsilon(n, n')\eta$.

In our method – with $M_\omega$ and $M_{br}$– weak dependence on heavy quark mass, similar results for charmonium and bottomonium.

This is in contrast to Multipole Expansion method (MEM), where prediction is

$$\frac{\Gamma(\Upsilon(2,1)\eta)}{\Gamma(\psi(2,1)\eta)} \cong 2.5 \cdot 10^{-3} \quad \text{and} \quad \frac{\Gamma(\Upsilon(3,1)\eta)}{\Gamma(\psi(2,1)\eta)} = 1.3 \cdot 10^{-3} \tag{30}$$

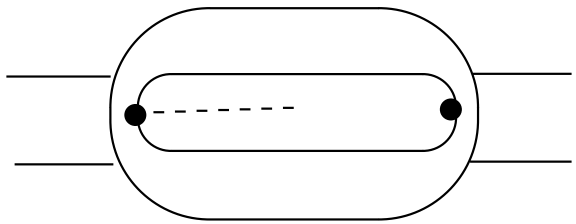
also (Voloshin, Zakharov) in MEM it is predicted that

$$\Gamma_\eta/\Gamma_{\pi\pi} \sim \frac{p_\eta^3}{(\Delta M)^7}, \quad \Delta M = M(\Upsilon(nS)) - M(\Upsilon(n'S)).$$

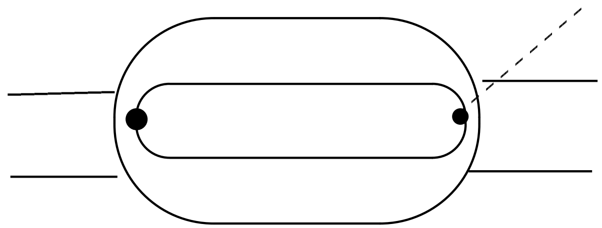
Both predictions do not agree with experiment (BaBar (2008))

$$\frac{\Gamma(\Upsilon(4,1)\eta)}{\Gamma(\Upsilon(4,1)\pi^+\pi^-)} = 2.41 \pm 0.40 \pm 0.12. \tag{31}$$

In our formalism one has graphs



( a )



( b )

The amplitudes are

$$\mathcal{M} = \mathcal{M}_\eta^{(1)} + \mathcal{M}_\eta^{(2)}; \mathcal{M}_\eta^{(i)} = \mathcal{M}^{(i)}_{B_s B_s^*} - \mathcal{M}^{(i)}_{BB^*}, i = 1, 2 \tag{32}$$

$$\mathcal{M}_\eta^{(1)} = \int \frac{J_n^{(1)}(\mathbf{p}, \mathbf{k}) J_{n'}(\mathbf{p})}{E - E(\mathbf{p})} \frac{d^3\mathbf{p}}{(2\pi)^3}, \tag{33}$$

$$\mathcal{M}_\eta^{(1)} = e_{ii'l} k_l \left( \frac{1}{\omega_s^3} \mathcal{L}_s^{(1)} - \frac{1}{\omega^3} \mathcal{L}^{(1)} \right) \tag{34}$$

$$\Gamma_\eta = \frac{1}{3} \sum_{i,i'} |\mathcal{M}|^2 d\Phi = \frac{2k^2}{3} d\Phi \left| \left( \frac{\mathcal{L}_s^{(1)}}{\omega_s^3} - \frac{\mathcal{L}^{(1)}}{\omega^3} \right) + \left( \frac{\mathcal{L}_s^{(2)}}{\omega_s^3} - \frac{\mathcal{L}^{(2)}}{\omega^3} \right) \right|^2 \tag{35}$$

Here $\omega_s, \omega$ are average energies of $s$-quark and $u, d$ quark in $B_s$ or $B$ meson respectively. Calculation with relativistic QCD string Hamiltonian yields $\omega_s = 0.639$ Gev, $\omega = 0.587$ GeV when $\omega_s = \omega$ one has $\Gamma_\eta = 0$. results are given in Table

**Table 3.** Values of $\Gamma_\eta(n,n')$ (in keV) calculated using Eq. (**??**) *vs* experimental data $\Gamma_\eta^{\exp}(n,n')$ (in keV).

| $(n,n')$ | 2,1 | 3,1 | 4,1 | 5,1 |
|---|---|---|---|---|
| $\frac{\Gamma_\eta(n,n')}{\left(\frac{M_{br}}{f_\pi}\right)^2\left(\frac{M_\omega}{2\bar\omega}\right)^2}$ | $5.0 \cdot 10^{-2}$ | 2.9 | 1.81 | 7.04 |
| $\Gamma_\eta^{\exp}(n,n')$ | $(0.8 \pm 0.3) \cdot 10^{-2}$ [4] | - | $4.02 \pm 0.6$ [12] | - |

Conclusion: Our method gives a reasonable prediction for $\gamma_\eta$ (up to a factor $\sim 2$ ). Note $\Gamma_\eta(5,1) \approx 7$ keV (large!)

# Strong channel interaction and new XYZ states. Project.

Channel coupling is due to $X \to D\bar{D} \to X'$ where $X$ is any heavy Quarkonia state.

One can write interaction kernel (potential, but energy dependent)

$$\text{channel 1}, \quad X_n, \quad \text{discrete spectrum}$$

$$\text{channel 2}, \quad D_{n_2}\bar{D}_{n_3} \quad \text{two body free}$$

General Theory of CC interaction is in Badalian, Kok, Polykarpov, Yu.S. Phys. Rept. '83

$CC$ Interaction in channel 2.

$$V_{212}(\mathbf{r}, \mathbf{r}', E) = \frac{M_\omega^2}{N_c} \sum_n \frac{J_n(\mathbf{r}) J_n(\mathbf{r}')}{E_n - E}$$

$J_n(\mathbf{r})$ – overlap integral

$$J_n = J_{nn_2n_3} = (\Psi_n / \psi_{n_2} \psi_{n_3})$$

$||V_{212}||$ is large and able to support its own bound states!

# Equation for level positions and widths

$$g^{(2)}_{mn}(E) = \frac{m_\omega^2}{N_c} \int \frac{J^*_m(\mathbf{k}) J_n(\mathbf{k}) d^3\mathbf{k}}{\frac{\mathbf{k}^2}{2M_2} - E(2\pi)^3}$$

$$det(\delta_{mn} - \frac{g^{(2)}_{mn}(E)}{E - E_n}) = 0$$

For one level new position $E^*_n$

$$E^*_n = E_n - g^{(2)}_{nn}(E)$$

$$\Gamma_n = 2Jm g^{(2)}_{nn}(E_n)$$

Was used above for $\Upsilon(nS) \to B\bar{B}$.

**Project:**

1. Study of new $X, Y, Z$ states as possible $CC$ resonances.
2. Calculation of $CC$ energy shifts and widths of legitimate $Q\bar{Q}$ resonances.
3. Calculation of $\pi Q\bar{Q}$ and $\pi\pi Q\bar{Q}$ resonances. (The same interaction kernel!)

# Conclusions

1. interaction of $X(Q\bar{Q})$ with heavy-light mesons is strong, $M_\omega \approx 1$ GeV
2. Interaction of pions with $X(Q\bar{Q})$ is weaker, $M_{br} \sim 0.1$ GeV, but still strong.
3. Decays $X \to B\bar{B}, ...$ are well described by theory with $\left(\frac{M_\omega}{2\omega}\right)^2 \approx 0.6$.
4. Dipion spectra are well reproduced by theory when $\left(\frac{M_{br}}{f_\pi}\right)^2 = 1.6$.
5. Single $\eta, \pi$ and $\gamma$ are predicted in the same mechanism.
6. Framework for multichannel calculations is created.