PROPOSAL FOR POLARIZED PROTONS ACCELERATION AT U-70

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Abstract

A possibility to accelerate polarized proton at the synchrotron U-70 is considered. Estimations of the spin resonances are given in the energy range $2.5 \div 70\, GeV$. To suppress depolarizing effects of spin resonances, a scheme of three partial Siberian snakes is suggested. Each snake consists from helical magnets with $4.5\, T$ magnetic field. The scheme provides the adiabatic spin flip at imperfection resonances and suppresses all intrinsic resonances by quite realistic beam emittance and magnets misalignments $\pm 0.5\, mm$.

1 Introduction

At present moment there is a reach experience in an acceleration of polarized protons in synchrotrons. But in each concrete case one have to choose more appropriate equipment and to solve problems how to implement these parts in the existing machine lattice. First attempt to consider this problem for U-70 have been done in 1984 year. However, proposed measures have demanded big enough efforts to enlarge one straight section by displacements of a number of dipole magnets. First of all, let's remind main approach to description of spin motion in accelerators. Its good known that particles motion in modern accelerators is described with extremely high accuracy by the semi-classical approach. But for the spin, these is no quasi-classical limit when orbital quantum number are large. Even at highest energies, an electron, or a proton remains in eigen states "up" or "down" as particles with spin $\frac{1}{2}$ (in units of $\hbar$). The quantum operator for this spin is $\hat{\boldsymbol{S}} = 1/2 \cdot \hat{\boldsymbol{\sigma}}$, where
$\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z) = \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$ - Pauli matrixes.

Follow the Ehrenfest's theorem we determine in the particle rest frame a classical vector of spin in any state $|\psi\rangle$ as a quantum average of spin operator $\boldsymbol{S} = \langle \psi^\dagger | \hat{\boldsymbol{\sigma}} | \psi \rangle$. This vector $\boldsymbol{S}$ precess in the rest frame around magnetic field $\boldsymbol{B}_c$ together with particle magnetic moment $\boldsymbol{\mu} = q\boldsymbol{S}$:

$$\frac{d\boldsymbol{S}}{d\tau} = \boldsymbol{\Omega}_c \times \boldsymbol{S}.$$

Spin precession frequency $\boldsymbol{\Omega}_c = -(q_0 + q')\boldsymbol{B}_c$, where $q_0 = \frac{e}{m}$ and q' are normal and anomalous parts of gyromagnetic ratio q. (We shall take later $c = \hbar = 1$).

A relativistic generalization of this spin motion equation to a laboratory frame have been done in different ways by many authors (see, for example, [1]). The most easy-to-use for accelerator applications view can be presented in the next form:

$$\boldsymbol{S}' = \frac{d\boldsymbol{S}}{d\theta} = \boldsymbol{W}(\theta) \times \boldsymbol{S} \tag{1}$$

with

$$\boldsymbol{W}(\theta) = -\frac{q_0}{\gamma}\Big[\Big(1+\gamma\, a\Big)\,\boldsymbol{B}_\perp + (\mathit{1}+a)\,\boldsymbol{B}_\parallel + \Big(\frac{\gamma}{\gamma+\mathit{1}} + \gamma\, a\Big)\,\boldsymbol{E}\times\boldsymbol{V}\Big].$$

Here we decomposed the magnetic fields in two projections $\boldsymbol{B}_\parallel$ and $\boldsymbol{B}_\perp$ (along and perpendicular to particle velocity $\boldsymbol{V}$ and introduce instead of time t a generalized accelerator azimuth θ and so called *magnetic anomaly* of a particle $a = \frac{q\prime}{q_0}$. For protons: $a_p = 1.792847351 \pm 2.8 \cdot 10^{-8}$.

Similar to the orbital motion we share the spin precession frequency in two parts:[2] $\boldsymbol{W}(\theta) = \boldsymbol{W}_0(\theta) + \boldsymbol{w}$. The periodical part $\boldsymbol{W_0}(\theta+2\pi) = \boldsymbol{W}_0(\theta)$ gives spin rotations by fields on the CO , whereas $\boldsymbol{w}$ is a small distortion ($|\boldsymbol{w}| \ll |\boldsymbol{W}_0|$), connected with momentum off particle oscillations. It's evidently, the solution of the spin motion equation (1) at any azimuth θ with $\boldsymbol{W}(\theta) = \boldsymbol{W}_0(\theta)$ is a periodical unit vector $\boldsymbol{n}_0(\theta+2\pi) = \boldsymbol{n}_0(\theta)$, which is the spin precession axis. A spin rotation around $\boldsymbol{n}_0(\theta)$ by an angle ϕ substitutes all spin rotations by arbitrary local fields along the CO.

There is a simple approach to find the spin closed orbit $\boldsymbol{n}_0(\theta)$ using the SU(2) matrixes. It's known, that a spin rotation on the i-th part of the orbit is described by the matrix: $T_i = I \cdot cos\frac{\phi_i}{2} - i(\boldsymbol{n_i}\cdot\boldsymbol{\sigma})sin\frac{\phi_i}{2}$, where $\boldsymbol{n_i}$ is a local precession axis.

Now, if we know the spin transfer matrix after one particle turn, which is a product of local rotations: $T = T_m \cdot T_{m-1} \cdots T_2 \cdot T_1$, then it's easy to find:

$$\boldsymbol{n}_0 = \frac{i}{2\pi sin\pi\nu} tr(\boldsymbol{\sigma}\cdot T); \qquad cos\pi\nu = \frac{1}{2}tr(T), \tag{2}$$

where we defined (similar to the betatron tunes) a spin tune $\nu = \frac{\phi}{2\pi}$, which evidently is a ratio $\boldsymbol{W}_0$ to the Larmor frequency:

$$\boldsymbol{\omega} = -\frac{q_0}{\gamma}\Big[\boldsymbol{B}_\perp + \frac{\gamma^2}{\gamma^2-1}\,\boldsymbol{E}\times\boldsymbol{V}\Big]. \tag{3}$$

Two others (perpendicular to $\boldsymbol{n}_0$) eigen solutions of the spin equation are complex vectors $\boldsymbol{\eta}$ end $\boldsymbol{\eta}^*$ rotating clockwise and contra-clockwise around $\boldsymbol{n}_0$ with the spin tune: $\boldsymbol{\eta}(\theta+2\pi) = \boldsymbol{\eta}(\theta)\,e^{-i\nu\theta}$.
A precession axis for spin of momentum off particles is slightly differs from $\boldsymbol{n}_0$ and can be found in the form $\boldsymbol{n} = \sqrt{1+|C|^2}\,\boldsymbol{n}_0 + Re(iC\boldsymbol{\eta}^*)$; $|C| \ll 1$. Putting this $\boldsymbol{n}$ into (1) we come in the linear approximation to the short-cut equation:

$$C' = w_\perp = (\boldsymbol{w}\cdot\boldsymbol{\eta}^*)\,. \tag{4}$$

2 Spin resonances

In usual case of a machine with the vertical guiding field $K_z = \frac{B_z}{\langle B_z\rangle}$ the spin precession axis $\boldsymbol{n}_0$ coincides everywhere with the unit vector along the guiding field: $\boldsymbol{n}_0 = \boldsymbol{e}_z$ and $\boldsymbol{\eta} = (\boldsymbol{e}_x - i\boldsymbol{e}_y)\,e^{-i\nu_0\tilde{\theta}}$, where $\tilde{\theta} = \int_0^\theta K_z d\theta$. From (1) and (3) we see, that spin tune in the accelerator frame $(\boldsymbol{e}_x;\, \boldsymbol{e}_y;\, \boldsymbol{e}_z)$ is equal to $\nu = \nu_0 = \gamma \cdot a$. Linear dependence of the spin tune on particle energy creates many troubles while acceleration.

Focusing elements are unavoidable at any circular accelerator the same as the betatron oscillations. It's appeared immediately as a distortion for the spin motion. A vertically deviated particle meets the radial component of the focusing magnetic fields. Using the particle motion equation this distortion can be described by $w_\perp = \nu_0 z'' = \nu_0 g_z z$. Putting that into (4) one finds, that $\boldsymbol{n}$ oscillates around $\boldsymbol{n}_0$ with the betatron tunes ν_z. In the resonance case $\nu = \nu_k = k \pm \nu_z$ (so called "intrinsic resonances") spin will rotates around the horizontal axis with a precession frequency w_k , which is the resonance strength:

$$|w_k| = |A_z| \frac{\nu_0}{2\pi} \oint g_z |f_z| e^{i(\nu_k \mp \nu_z)\tilde{\theta}} d\theta,$$

where we used the Floke form for the solution of z-motion equation: $z = A_z f_z + c.c.$. One can see, that the strength of the intrinsic resonances enhances with the particle energy and depends on the vertical oscillation amplitude A_z.

Other spin distortion is connected with vertical deviations of the CO, caused by radial imperfection fields $K_x = \frac{B_x}{\langle B_z \rangle}$. In this case putting into (4) the forced periodical part of the vertical motion Z_s we get strengths of imperfection resonances $\nu = k$:

$$|w_k| = \frac{\nu_0}{2\pi} \oint Z_s'' e^{-ik\tilde{\theta}} d\theta = \frac{\nu_0}{2\pi} \oint K_x F_3(\theta) e^{-ik\tilde{\theta}} d\theta \,,$$

where $F_3(\theta) = \frac{dn}{dz'}$ is a spin response function, that reflects a sensitivity of $\boldsymbol{n}$-vector to vertical kicks.[6]

These imperfection spin resonances take place in each 523.342 MeV for protons and their strengths increase with beam energy faster than the intrinsics ($F_3 \sim \gamma$). Most strong imperfection resonances follow also the machine periodicity $k = mP$ (m-integer). But unlike to the intrinsic resonances one can adjust the vertical closed orbit to minimize w_k up to level depending on his experimental technics.

3 Spin resonance crossing

Since the spin tune is proportional to the energy, spin resonance crossings are unavoidable while an acceleration. A gap between two imperfection resonances is equal to 523.342 MeV for protons. The intrinsic resonances are located symmetrically around each imperfection resonance. So, the acceleration of polarized particles looks as very complicate issue.

In the simplest case of a separate resonance $\nu = \nu_k$ with a strength w_k a final polarization ζ_f after one crossing with tune rate $\dot{\delta} = \frac{d(\nu - \nu_r)}{dt}$ will be differs from the initial value ζ_0:

$$\zeta_f = \zeta_0 \left(2e^{-\Psi} - 1 \right), \tag{5}$$

where $\Psi = \frac{\pi w_k^2}{2\dot{\delta}}$ is a spin phase advance in the resonance zone (tune $\delta \sim w_k$).[3]

When $\Psi \ll 1$ (fast crossing) polarization loss is small: $\delta\zeta \simeq \zeta_0 \Psi$. More interesting is opposite case: $\Psi \gg 1$, which leads to a spin flip ($\zeta_F \simeq -\zeta_0$) with an exponentially low depolarization: $|\delta\zeta| = 2\zeta_0 e^{-\Psi}$.

Both situations are widely used in accelerator practice. A suppression of the resonance strength (orbit corrections) or increasing tune rate (tune jump) lead to the fast crossing and vice versa an artificial resonance enhancement helps to safely reverse the polarization. It's appeared, that so called partial Siberian snake is more convenient way for an enlargement of imperfection resonances.[4],[5]

However, it's evident, that with increasing of the energy these games become more and more complicate.

4 Siberian snakes

To suppress spin resonances totally in 1976 it was suggested to introduce in machine lattice a special chain of magnets, which will rotate spin by 180 degrees by pass [6]. How it easy to see, in this case the spin tune $\nu = 1/2$ independently on the beam energy and the resonance condition does not more arise. This approach was called as Siberian snake. It can be arranged in different magnet combinations, which rotates spin around any axis. At that the periodical precession axis $\boldsymbol{n}_0$ at opposite to the snake azimuth lies in the horizontal plane with the same angle to the CO as the snake axis.

The simplest snake is one solenoid with the longitudinal integral field proportional to the beam energy; (37 Tm for 10 GeV protons). Some disadvantage of the solenoidal snake is a x-z coupling, introduced by the longitudinal field. There are a number of schemes with combinations of solenoids and skew quads, that don't excite the coupling outside the insertion. But such schemes require, as rule, a long and free from other equipments straight section.

More compact Siberian snakes can be designed from dipole magnets by combinations of horizontal and vertical bends. How it's seen from the expression (1), spin rotations by transverse fields don't depend on energy for high energy particles, whereas the action on spin by longitudinal fields is inverse proportional to the energy. But main disadvantage of transverse fields is big orbit deviations and, as a consequence, a wide aperture of the magnets and so on. Many years it has restricted considerably applications of the transverse fields for snake constructions.

However, these problems can be minimized in the case of helical magnets.[7] A set of full twist helixes with mirror symmetry provides the compensation of orbit distortions outside the snake taking them very moderate inside the insertion. This advantage creates interesting possibilities to use helical magnets for both full snake and for partial one [8]. Moreover, such systems shown a great flexibility for an obtaining practically arbitrary snake axis. This fact is appeared very important for the acceleration of polarized protons at RHIC.[9]

5 Proton synchrotron U-70 and spin resonances

The proton synchrotron U-70 is operating successfully in Protvino since 1967 year (see Fig.1). The beam is accelerated in the proton linac Ural 30, then in the booster and injected into U-70 with the kinetic energy 1.5 GeV, where then its energy is increased during 2 seconds up to 70 GeV.

The synchrotron lattice has 12 periods FODO structure. Each period consists from 20 dipoles with alternative gradient. Optical functions and the structure of the one period are shown in the Fig.2

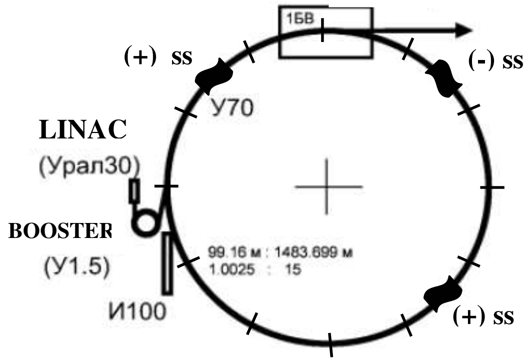

Figure 1: U-70 complex layout

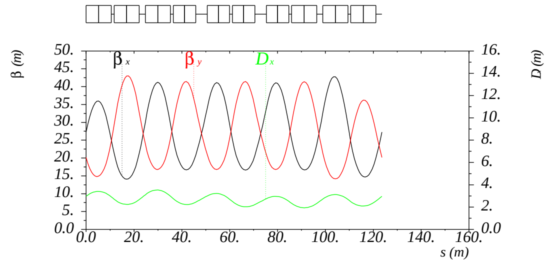

Figure 2: Optical functions of U-70

To understand the situation with depolarizing resonances at U-70 first of all an evaluation of intrinsic spin resonance strengths have been done for normalized vertical beam emittance $\epsilon_z = 10\pi\ mm \cdot mrad$ and betatron tunes $\nu_z = 0.8$ and $\nu_x = 0.77$. The ASPIRRIN computer code was used for that.[10]

As it was expected, the calculations shown, that for high energy ($\nu_0 \simeq 100$) resonances $\nu_0 = kP \pm \nu_z$ (P is a machine periodicity) grow up to $|w_k| \sim (0.2 \div 0.4) \cdot \omega_0$ by $A_z \simeq 1mm$ (dotted lines in the Fig.3).

The same picture contains also results of estimations of imperfection resonances: $\nu_0 = kP$ (solid lines). At that, we assumed vertical displacements of any dipoles by $\pm 0.5\,mm$. It's seen, that before acceleration of polarized protons one has to pay a serious attention to a bending magnets alignment. At the present moment (after almost 40 years machine operation) geodesic measurement data give misalignment in one order magnitude worse than it's needed.

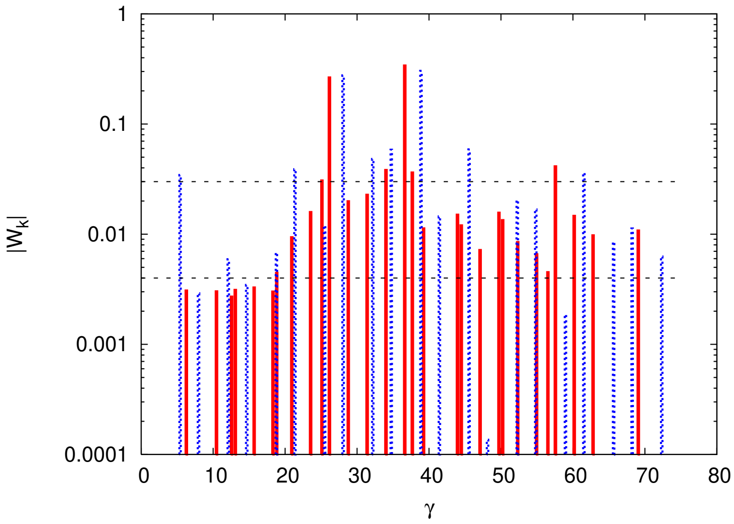

Figure 3: Spin resonances at U-70

Two horizontal dashed lines at the Fig.3 indicate two levels of the resonance strength, when according to the Froisart-Stora conditions (5), safe resonance crossings are possible. The lower line is the limit for the fast crossing. The adiabatic regime (spin flip) is available above the upper line. So, from these calculations one can conclude, that it's necessarily to search other approaches for successive acceleration of polarized protons at U-70.

6 Siberian snakes at U-70

The first attempt to modify U-70 to polarized proton acceleration have been taken yet at 1984. However, proposed scheme of the Siberian snake has required big machine changes and it was not accepted.

The present proposal suggests to install three identical partial snakes in three periods of U-70 lattice, as it's shown in the Fig.1. Each snake will be inserted in one of two 4 meter straights between bending magnets (see the chain of magnets in the Fig.2).

An optimized snake design takes $3.5\,m$ and consists of 4 periods of two-thread helical coil (a period $l = 70\,cm$; aperture $15\,cm$). The super-conducting coil is wound between the iron poles and is surrounded by the 40 cm diameter iron yoke.

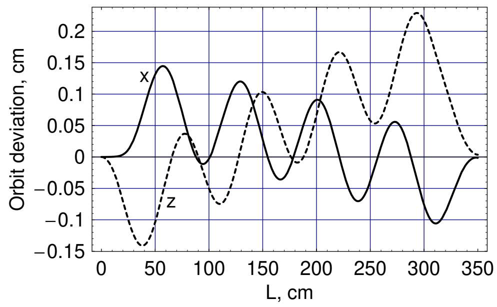

Figure 4: Orbit inside the snake

Parameters of the coil, iron and supply current have been optimized to reach on the axis the magnetic field up to $4.2\,T$. 3-D magnetic field calculations (including fringe fields) have been done by the MERMADE code . Then the differential equation of a particle motion was solved in the calculated nonlinear fields . Obtained particle trajectories inside the snake are shown in the Fig.4 for the energy $25\,GeV$. The orbit deviations are inverse proportional to the beam energy. They will achieve $2.5\,cm$ on the injection energy $E = 2.5\,GeV$. Since the spin resonances are weaker at the low energy, some modulation of the snake power supply can be apply to avoid aperture limitations during the injection.

To compensate some CO distortions, caused by the fringe fields, two dipole coils are wound at the snake edges. On this stage the dipole coils current was tuned for the CO adjustment (no orbit deviations outside the snake).

An estimation of the machine optics distortions shown, that three snakes together disturb a little bit the optical functions. To vanish this effect totally also thin quadrupole coils have to be added on the both edges of each snake.

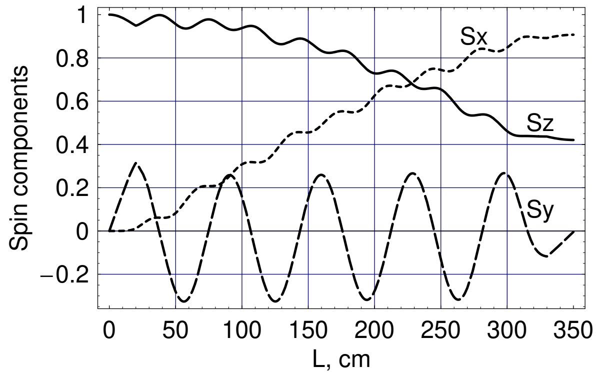

Figure 5: Spin rotation inside the snake

At the next step a solution of the BMT-equation for spin rotations was found in the helical fields along the determined particle pass through the snake. Parameters of the snake design have been optimized to rotate spin by 60 degrees. Spin vector components behavior along the snake is given in the Fig.5 for initially vertical spin.

From the Fig.5 it's seen, that the snake axis is practically longitudinal. It means, that spin SU(2) transfer matrix through the snake is equal to $T_S = I \cdot cos\frac{\pi}{6} - i\sigma_y \cdot sin\frac{\pi}{6}$. Taking this fact into account it's easy to find the one turn spin map with three snakes "on". Follow the mention above approach we get a behavior of the spin closed orbit $\boldsymbol{n}_0$ and the spin tune ν versus the beam energy.

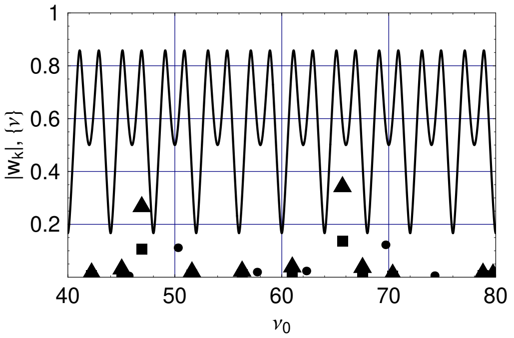

Figure 6: Spin tune with snakes and strong resonances

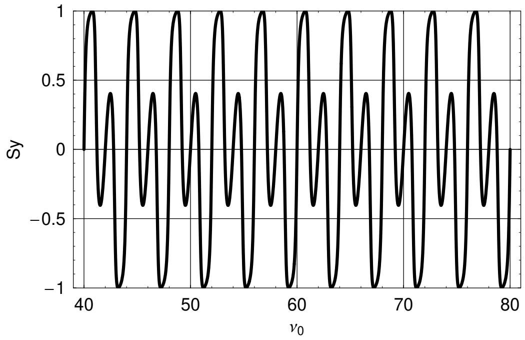

Figure 7: Vertical polarization versus ν_0

In the Fig.6 the solid line presents a fractional part $\{\nu\}$ of the spin tune against ν_0. Points in the Fig.6 correspond to the same most strongest spin resonances as in the Fig.3. One can see, that a strength of imperfection resonances, exited by the snakes, have only two fixed values: 0.18 and 0.5. It means, that resonant conditions for the weak intrinsic resonances ($|w_k| \leq 0.1$) can't appear in the proposed snake scheme, if the fractional parts of the betatron tunes don't exceed 0.2. Moreover, the scheme is tuned by a choice of the snake polarities to the case, when two more strongest intrinsic resonances ($|w_k| \simeq 0.3$ without the snakes) meet the spin tune 0.5.

The Fig.7 demonstrates spin behavior, caused by the snake induced imperfection resonances. We see a periodicity of full spin flips. For simplicity of spin manipulation while injection and extraction it's better to do that, when the polarization is vertical.

7 Conclusion

The present consideration shown, that the polarized proton acceleration at U-70 is possible with relatively moderate machine modification. The installation of three helical partial Siberian snakes in three period of machine structure can provide the suppression of all intrinsic resonances. To avoid undesirable interferences the snake and machine imperfection resonances the last have to be minimized by the magnet alignment with accuracy $\pm 0.5mm$.

The draft design of the helical magnet with field $B \simeq 4.5T$ is proposed. The estimations of parasitic snake influences on the particle dynamics has been estimated. Weak steering dipole and quadrupole coils have to be wound on the snake edges to keep the optics and orbit deviations inside the snake insertions.

Of course, to prove the full situation for polarized proton acceleration an additional spin and particle tracking simulation are needed.

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